Physics of Sawtooth and its control

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Outline

Introduction

- Motivation
- Stability of (1,1) mode in tokamak

Sawtooth period and amplitude (slow time scales)

- Sawtooth model for crash prediction
- Influence of the fast particles
- Control of fast particle stabilized sawteeth
- New control approaches

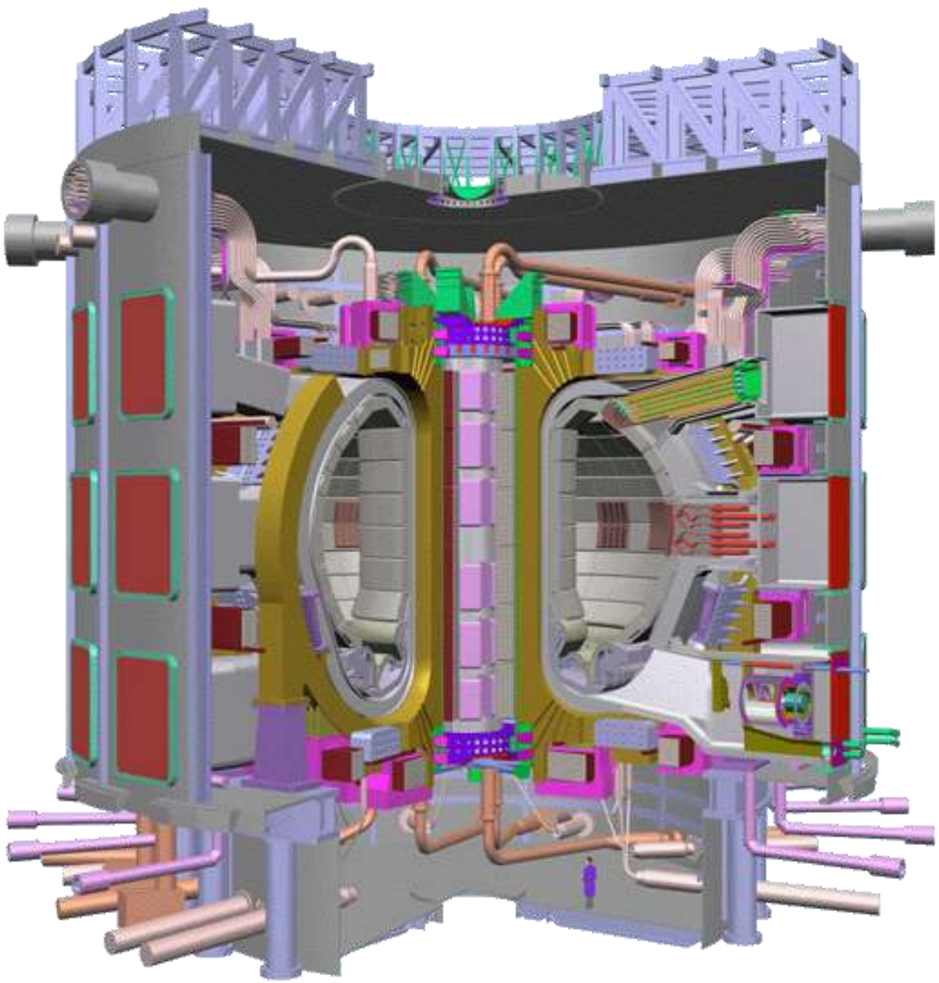
Crash event (fast time scales)

- Experimental observations
- Possible models
- Stochastic model
- Computation results

Conclusion

Why do we need to control sawteeth?

- **Problem with long Sawteeth?**



ITER will have large fusion-born alpha particle population in the core

↓

Long sawtooth periods

↓

Trigger NTMs and cause confinement degradation (reduced fusion power… etc), or worse disruption

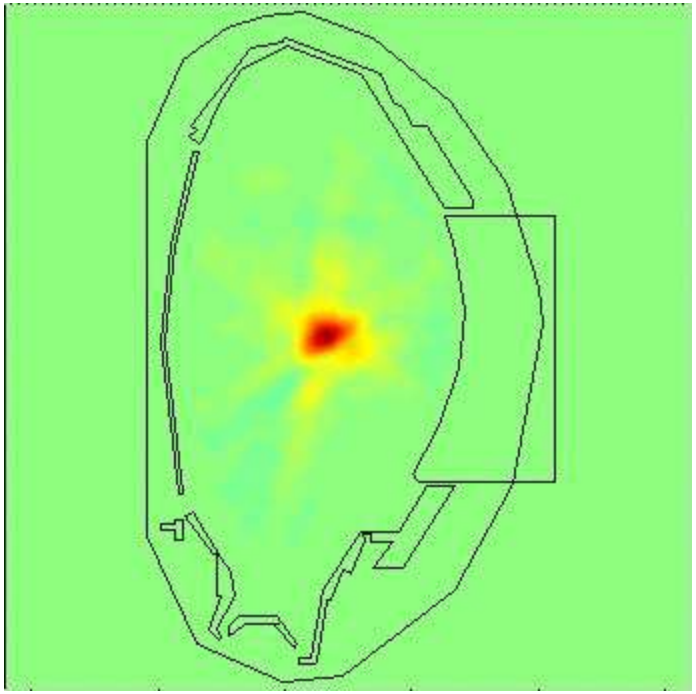
↓

Since NTMS can be at large amplitude very rapidily, a prevention approach recommended: keep sawteeth small and frequent

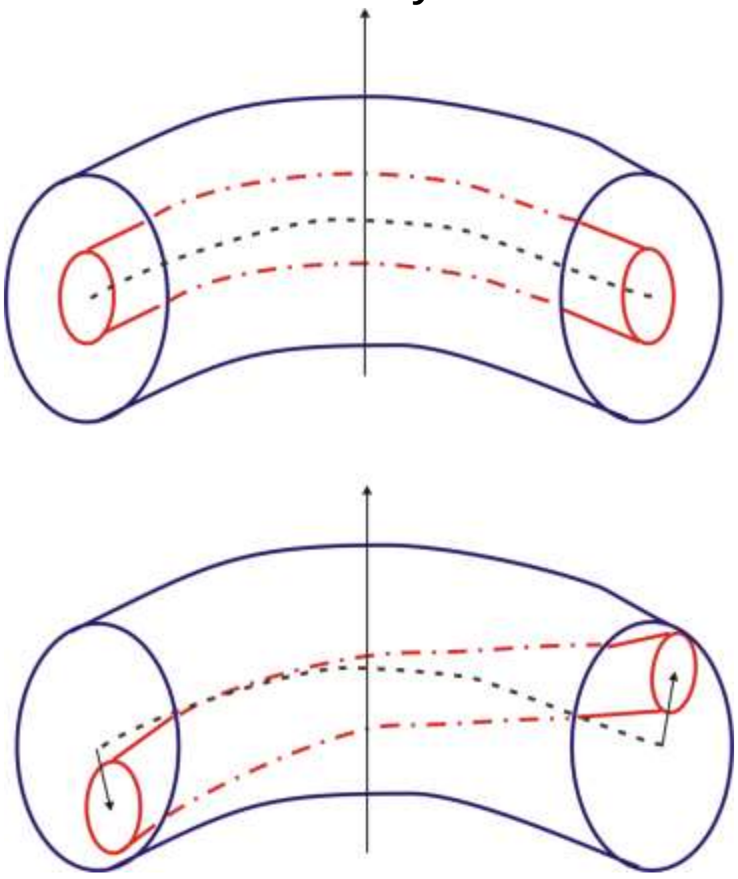
SXR tomography in ASDEX Upgrade

Topographic reconstruction of sawtooth crash in ASDEX Upgrade from soft X-ray cameras signals (maximal entropy method).

Discharge: 23074 t=3.07s (color scale is different for each time frame)

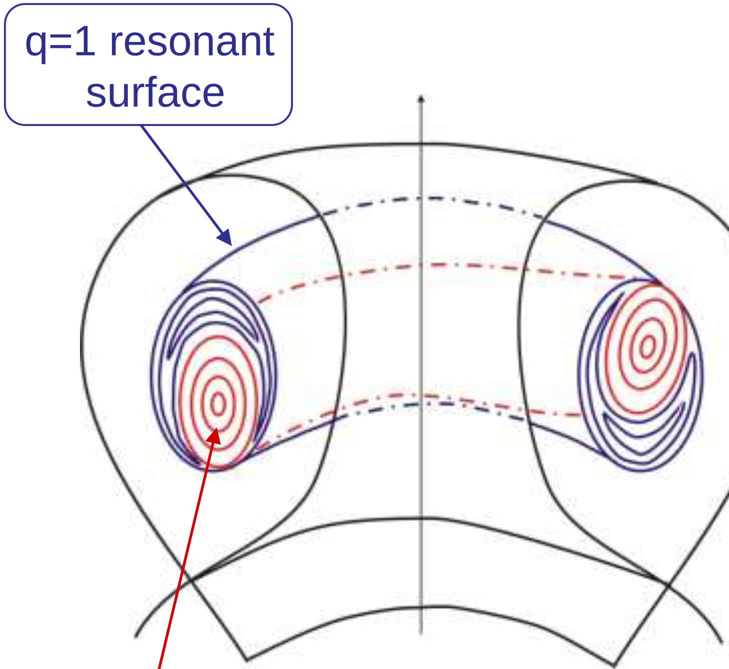


Internal Kink instability in a Tokamak.



A Tilt and Shift of the Core Plasma.

Sawteeth: internal (1,1) kink mode.



Energy principle: $\delta W(\boldsymbol{\zeta},\boldsymbol{\xi}) = -\frac{1}{2}\int \boldsymbol{\zeta}\cdot\boldsymbol{F}(\boldsymbol{\xi})\,d^3x$

Low β approximation, cylinder (good for current driven modes):

$$\boldsymbol{j}_\perp = \frac{\boldsymbol{B}\times\nabla p}{B^2} = \mathcal{O}(\varepsilon^2)\,, \qquad \varepsilon = \frac{a}{R_0} \ll 1$$

$$\boldsymbol{B} = B\begin{pmatrix} 0 \\ \frac{r}{qR_0} \\ 1 \end{pmatrix} + \mathcal{O}(\varepsilon^2), \quad \boldsymbol{j} = \frac{B}{R}\begin{pmatrix} 0 \\ 0 \\ \frac{1}{r}\left(\frac{r^2}{q}\right)' \end{pmatrix} + \mathcal{O}(\varepsilon^2)$$

$$\boldsymbol{\xi} = \boldsymbol{\xi}(r)e^{i(m\theta - n\phi - \omega t)} \qquad \boldsymbol{\xi} = \xi_r\hat{\boldsymbol{r}} + \xi_\theta\hat{\boldsymbol{\theta}} + \xi_\parallel\boldsymbol{b}$$

$$\delta W_{\rm cyl} = \pi^2\frac{B_0^2}{R_0}\int_0^a \left(|r\xi_r'|^2 + (m^2-1)|\xi_r|^2\right)\left(\frac{n}{m} - \frac{1}{q}\right)^2 r\,dr \; + \mathcal{O}(\varepsilon^4)$$

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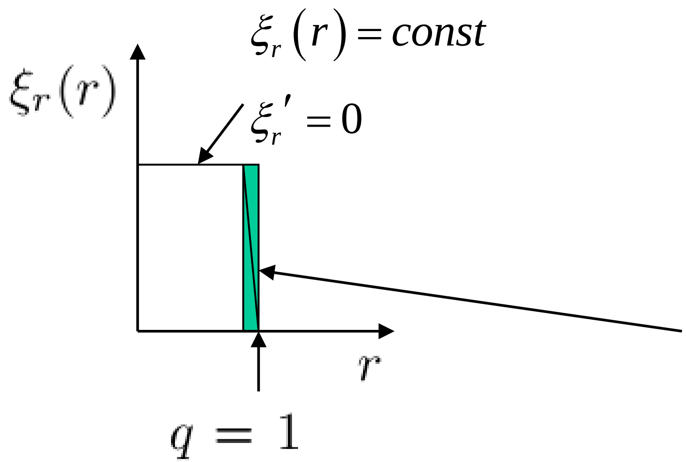
for m=1 $$\delta W_{m=1} = \pi^2 \frac{B_0^2}{R_0} \int_0^a r^3 |\xi_r'|^2 \left(1 - \frac{1}{q}\right)^2 dr$$



This function minimize the functional

δ width of the layer where $\xi_r(r)$ changes

$|\xi_r'|^2 = \mathcal{O}(1/\delta^2)$ $\qquad (1 - 1/q)^2 = \mathcal{O}(\delta^2)$

at $q \approx 1$

Small and large terms cancel in the integrand. It follows that the $\delta W_{m=1}$ vanishes because the integration interval itself has width δ

(1,1) internal kink mode is pressure driven

To understand stability of the (1,1) mode we have to compute next terms in the energy functional expansion $\delta W = \mathcal{O}(\varepsilon^4)$

If one takes into account now new terms (pressure gradient, perpendicular current, toroidal curvature, compressibility)

$$\delta W \approx 6\pi^2 \frac{B_0^2 r_1^4}{R_0^3} |\xi_r(0)|^2 \left[1 - q(0)\right] \left[\beta_{\text{crit}}^2 - \beta_p^2(r_1)\right]$$

Unstable only if $\beta_p > \beta_{crit}$!!
(1,1) internal kink mode is **pressure driven**!

$$\beta_{\text{crit}}^2 = \frac{13}{144}$$

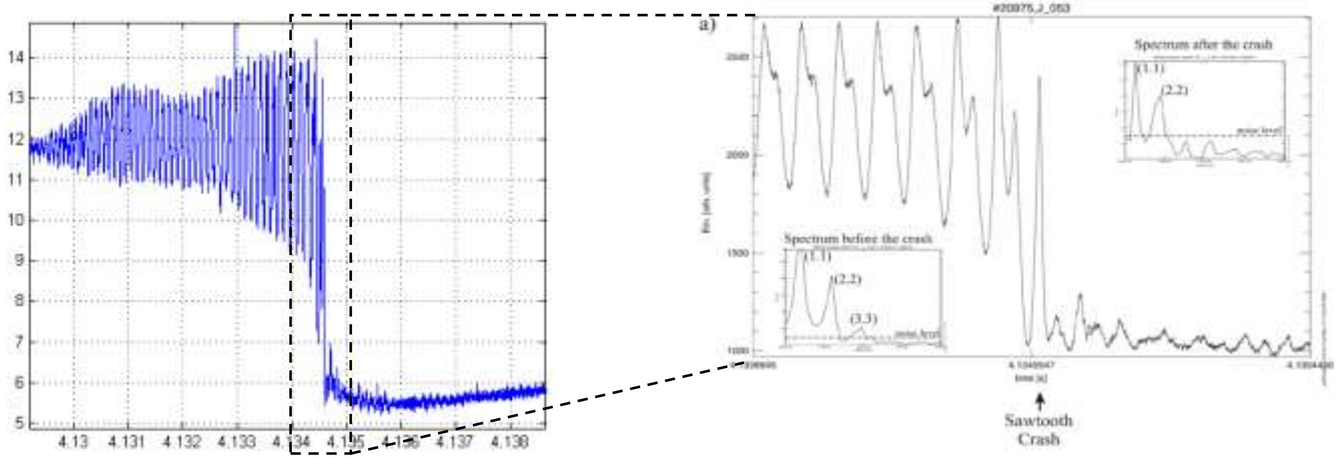
$$\beta_p(r_1) \equiv -2\frac{R_0^2 q^2}{B_0^2 r_1^4} \int_0^{r_1} p' r^2 dr$$

This is different to other internal kinks which are current driven in the simplest approximation

M.N. Bussac, R. Pellat, D. Edery, and J.L. Soule, "Internal kink modes in toroidal plasma with circular cross-section," *Phys. Rev. Lett.* **35**, 1638 (1975).

Rem.: Energetical particles can compete with small MHD contribution and lead to strong interaction with particles (discussed later)

Sawtooth



Understanding of the sawteeth

Slow time scale

- sawteeth period
- mixing radius
- amplitude

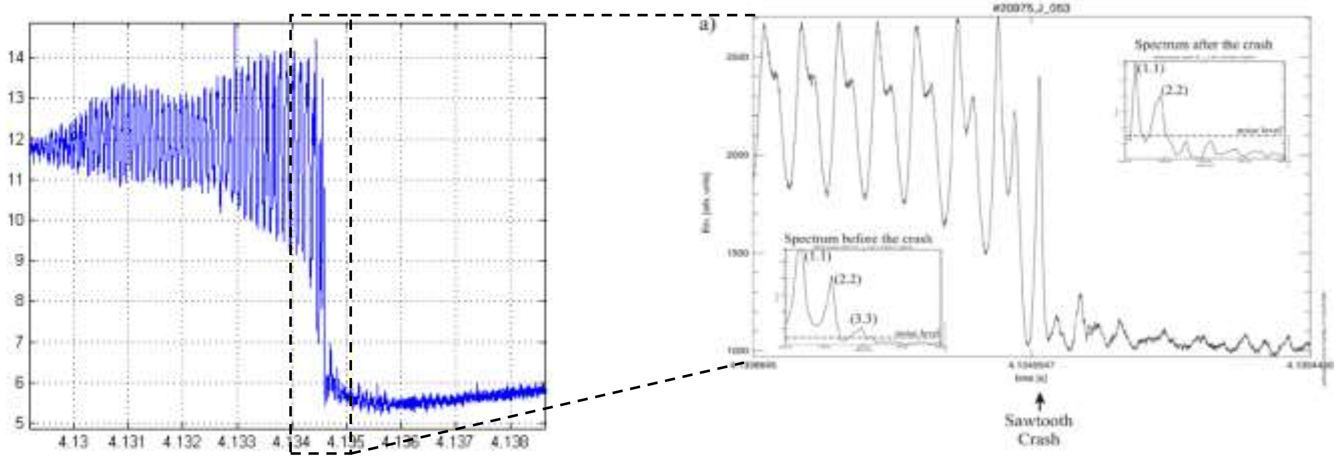
Especially important for ITER

Fast time scale

- mechanism of the crash

Understanding of the basic physics

Sawtooth



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Understanding of the basic physics

Porcelli model for sawteeth (slow time scale)

Porcelli-Boucher-Rosenbluth model for sawtooth period and amplitude

[Porcelli et al, PPCF, 38, 2163 (1996)]

Linear stability of the (1,1) mode is considered.

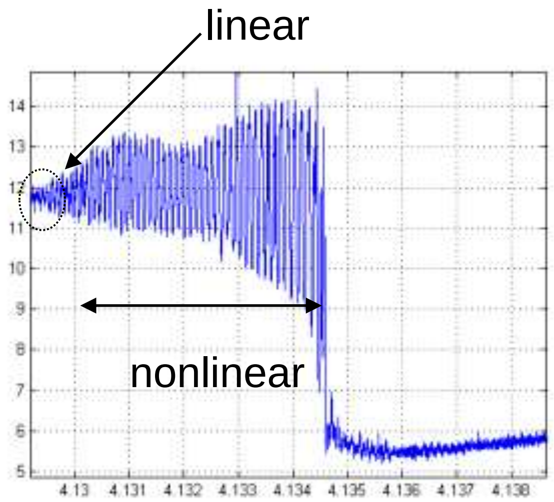
Sawtooth crashes are triggered by internal (1,1) kink modes if:

- *q(0) <1*
- resistive (1,1) becomes unstable or ideal (1,1) becomes unstable

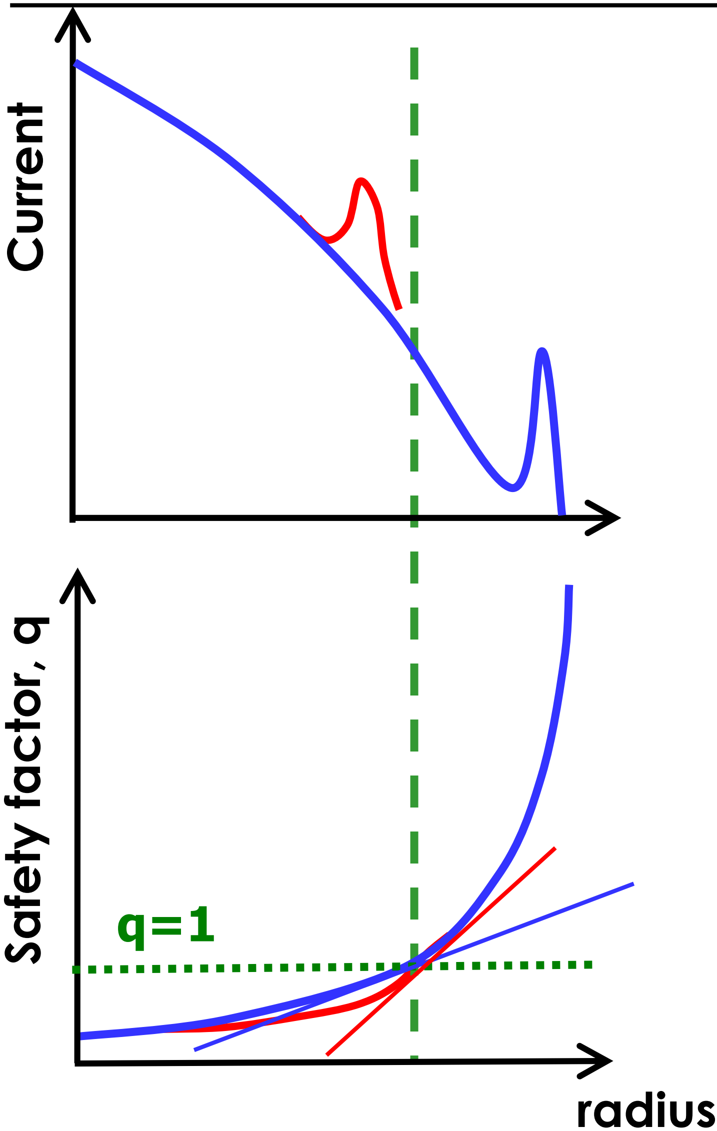
$$\frac{\delta \hat{W}}{s_1} < const$$

Model works in spite of the fact that (1,1) mode is strongly nonlinear !

(This is just a luck!)



Driving current changes the sawtooth stability



<u>Trigger condition:</u> $$\pi \frac{\delta \hat{W}}{s_1} < c_\rho \frac{\rho}{r_1}$$

A perturbation is applied to the initial current profile (inside q=1)

More current means more poloidal field, so q drops

q=1 is moved outwards

Magnetic shear at q=1, s_1 (~gradient of q) increases

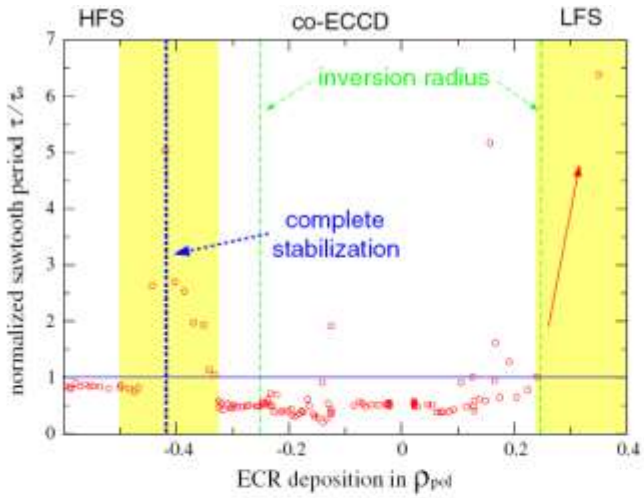
What else influence the sawteeth stability ?

$$\frac{\delta \hat{W}}{s_1} < const$$

Local current drive

Influence on the shear at q=1

<u>Example:</u> Change of shear at q=1 with co-ECCD and counter-ECCD in ASDEX Upgrade **[A.Mueck, PPCF, 2005]**



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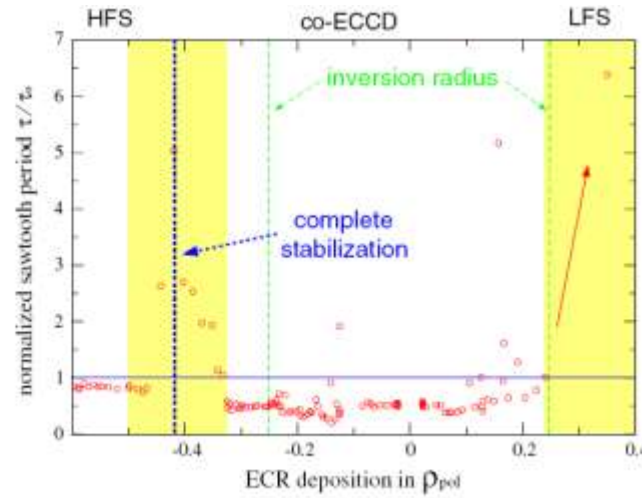


$$\frac{\delta\hat{W}}{s_1} < const$$

Influence on the mode stability δW

Toroidal rotation

Plasma shape

Heating deposition

Profiles

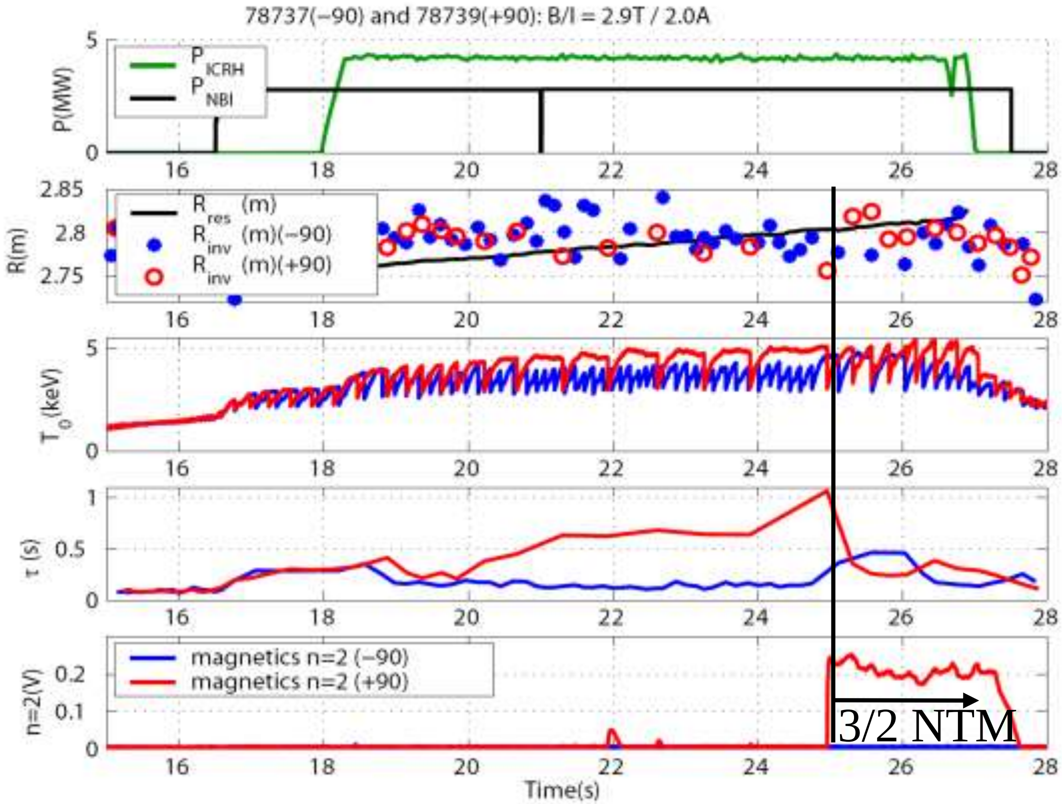
Fast particles

Relatively good understood

Fast particles are expected to be strongly stabilizing in ITER

Different ICRH heating of the ^{3}He minorities in JET

$2.9T < B < 2.96T$ (co-, cntr- propagating)



[Graves et al, NF 2010]

Current drive is negligible because we are influence on minority (no influence on s_1)

Main effect has kinetic nature (change δW)

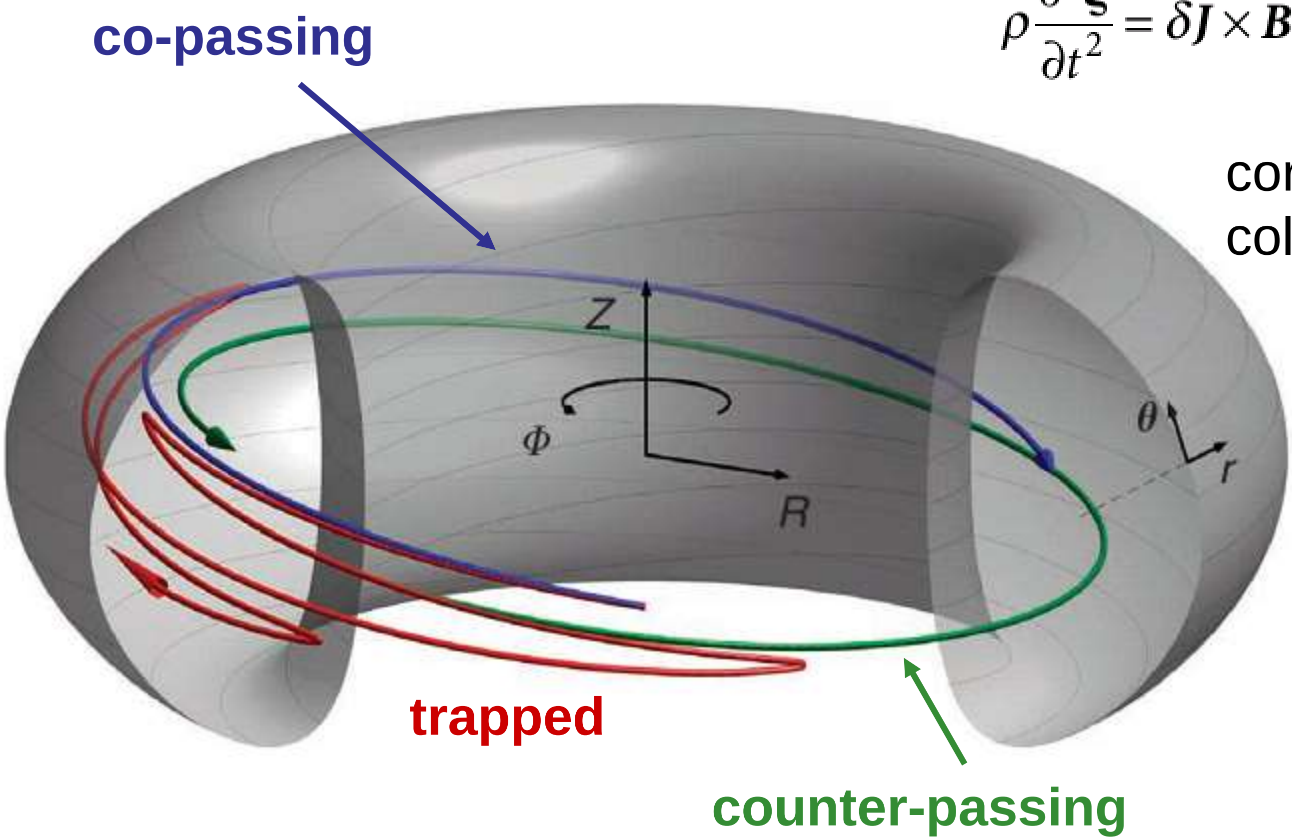
$$\frac{\delta \hat{W}}{s_1} < const$$

Different ICRH heating of the ^{3}He minorities in JET

$$\rho \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} = \delta \boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{J} \times \delta \boldsymbol{B} - \boldsymbol{\nabla} \delta P_c + \mathcal{F}_h(\delta \underline{\underline{P_h}})$$

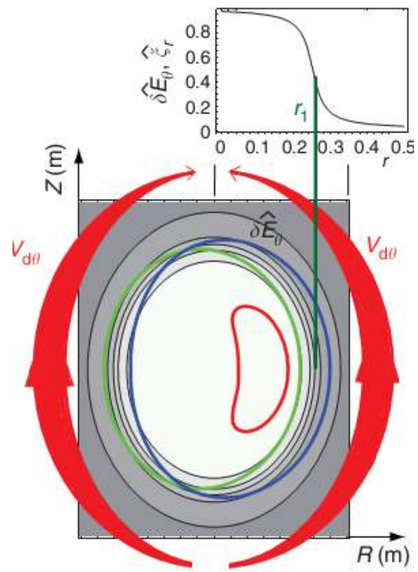
core collisional

hot collisionless



J.P. Graves, Nature, 2012

Different ICRH heating of the ^{3}He minorities in JET



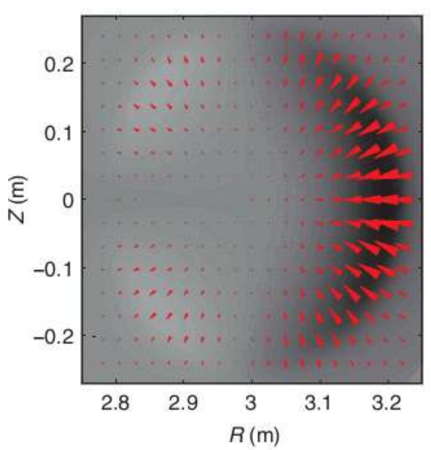
$$\rho \frac{\partial^2 \boldsymbol{\xi}}{\partial t^2} = \delta \boldsymbol{J} \times \boldsymbol{B} + \boldsymbol{J} \times \delta \boldsymbol{B} - \boldsymbol{\nabla} \delta P_c + \mathcal{F}_h(\delta \underline{\underline{P_h}})$$

core (collisional) force

hot ions (collisionless) force

Radial component of the normalized force points inward for trapped ions if displacement is outward. (Stabilizing effect from $\hat{\mathcal{F}}_h$!)

Physically this means conservation of third adiabatic invariant. (If low frequency perturbation is trying to change adiabaticaly the flux through these orbits, the orbits will tilt or shift in space in order to preserve this flux)



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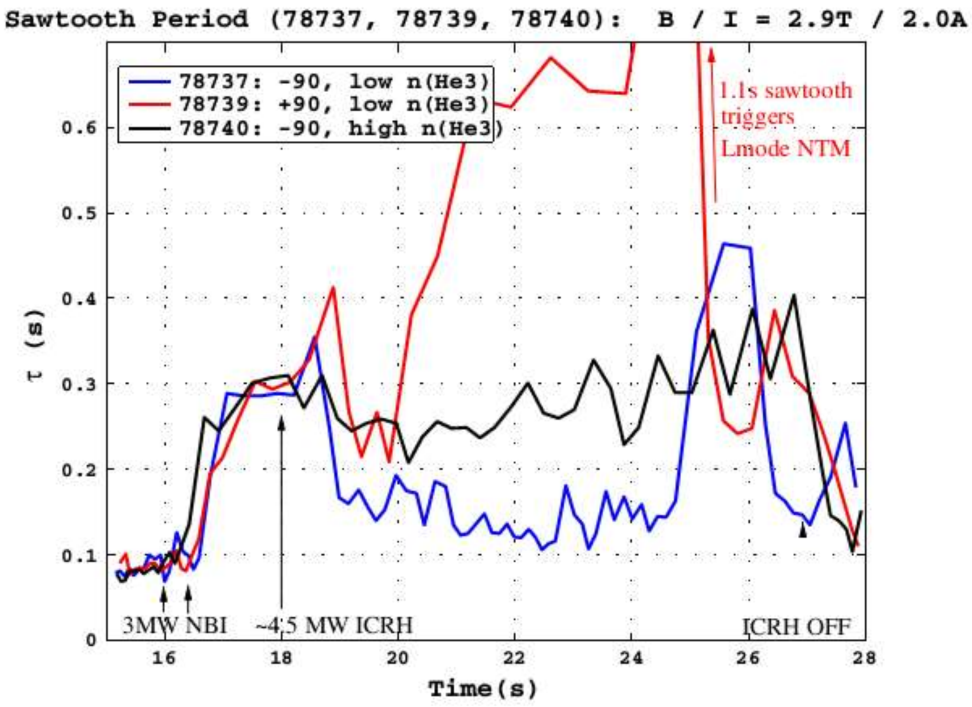


Figure 5. The sawtooth period for 78737 (−90° phasing, low concentration ^{3}He), 78740 (−90° phasing, high concentration ^{3}He) and 78739 (+90° phasing, shown also in figure 1).

The effect strongly depends from the ^{3}He concentration.

Thus, the main question is:

How to control the minority concentration in ITER and DEMO?

The strongest effect on mode stability is for a ^{3}He concentration of only 1%. When there is too much ^{3}He, the energy of the particles in the tail of the distribution becomes too low to have a strong effect on the kink mode whereas too little ^{3}He means that the absorbed power is low and the broader distribution function leads to increased fast ion losses.

Changes of the beam direction

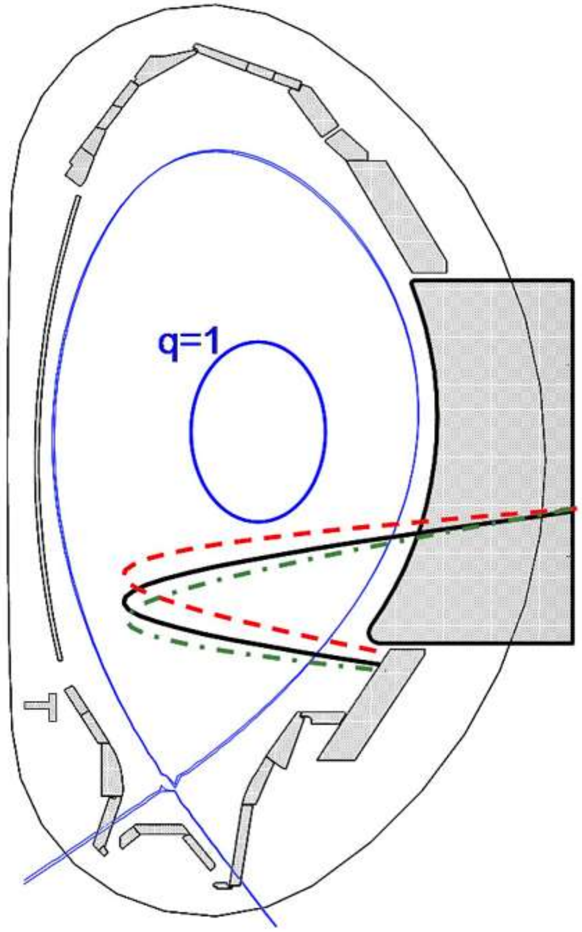
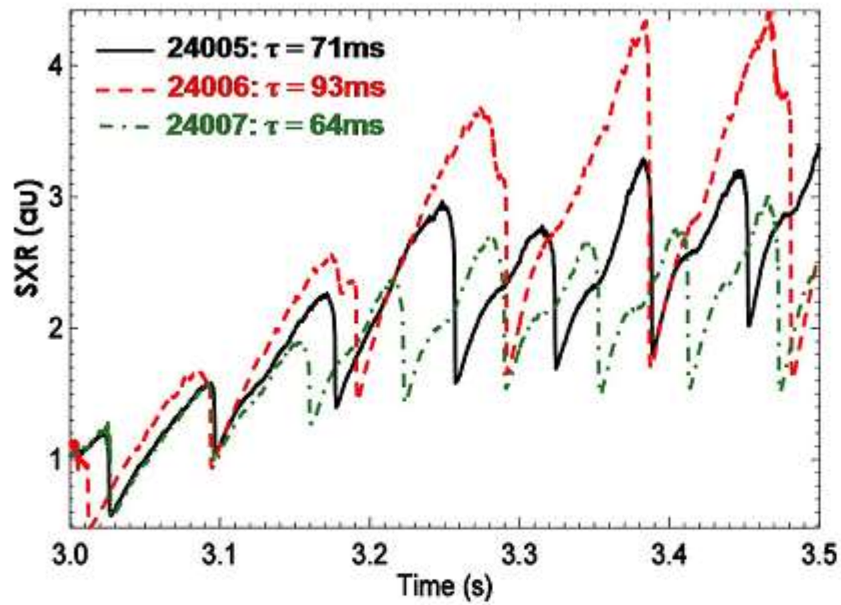


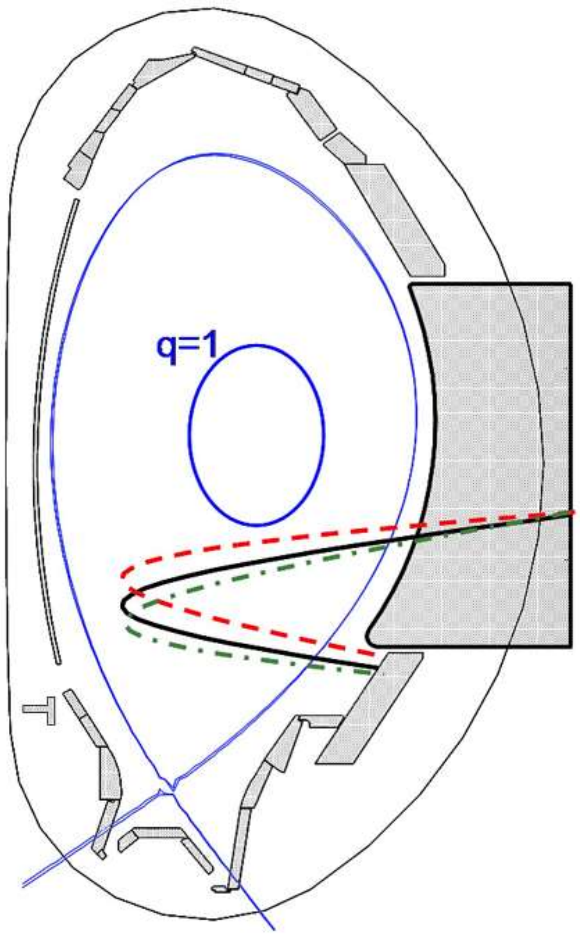
FIG. 1. (Color online) The beam trajectories of the off-axis PINI in ASDEX Upgrade as the PINI is tilted on its support. Also shown for comparison is the approximate position of the $q=1$ surface.



[I.T. Chapmen, V.Igochine, et.al.Nucl. Fusion 49 (2009)]
[I.T. Chapmen et.al. Physics of Plasmas 16 (2009)]

Position of the NBI with respect to q=1 surface is important

Changes of the beam direction



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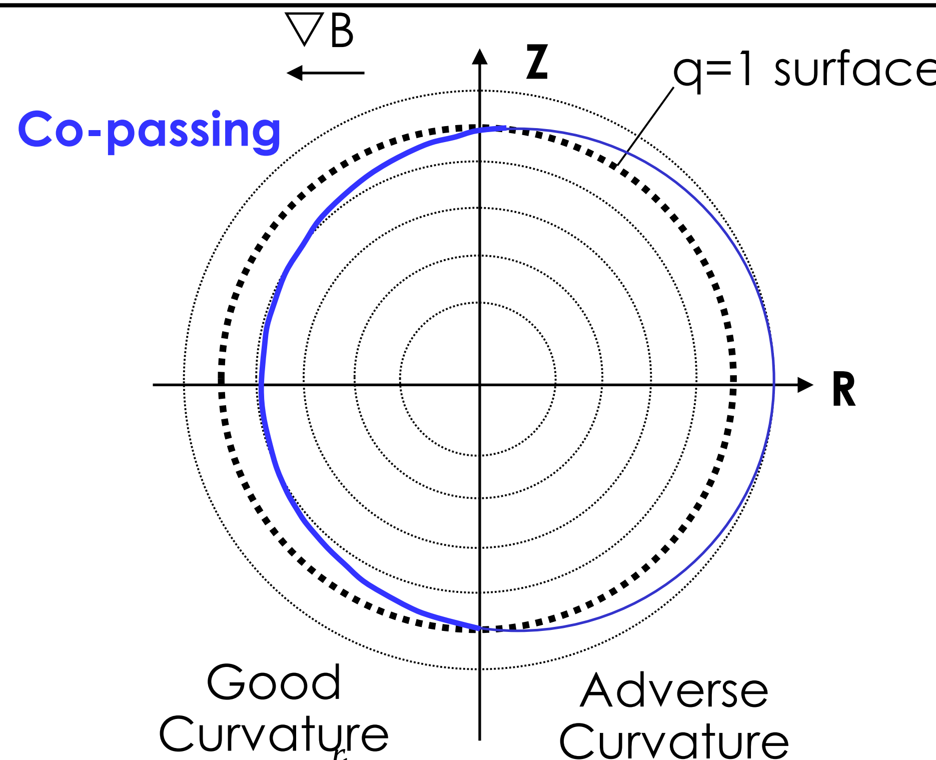


$$\delta W \sim -\int_0^{r_1} \left(\xi \cdot \nabla \langle P_h \rangle\right)\left(\xi \cdot \kappa\right) dr \neq 0$$

Co-pass, $\langle P_h \rangle' |_{r_1} < 0 \rightarrow$ stabilising

Co-pass, $\langle P_h \rangle' |_{r1} > 0 \rightarrow$ destabilising

[Graves, PRL, 2004; Chapman et al, PPCF, 2008]

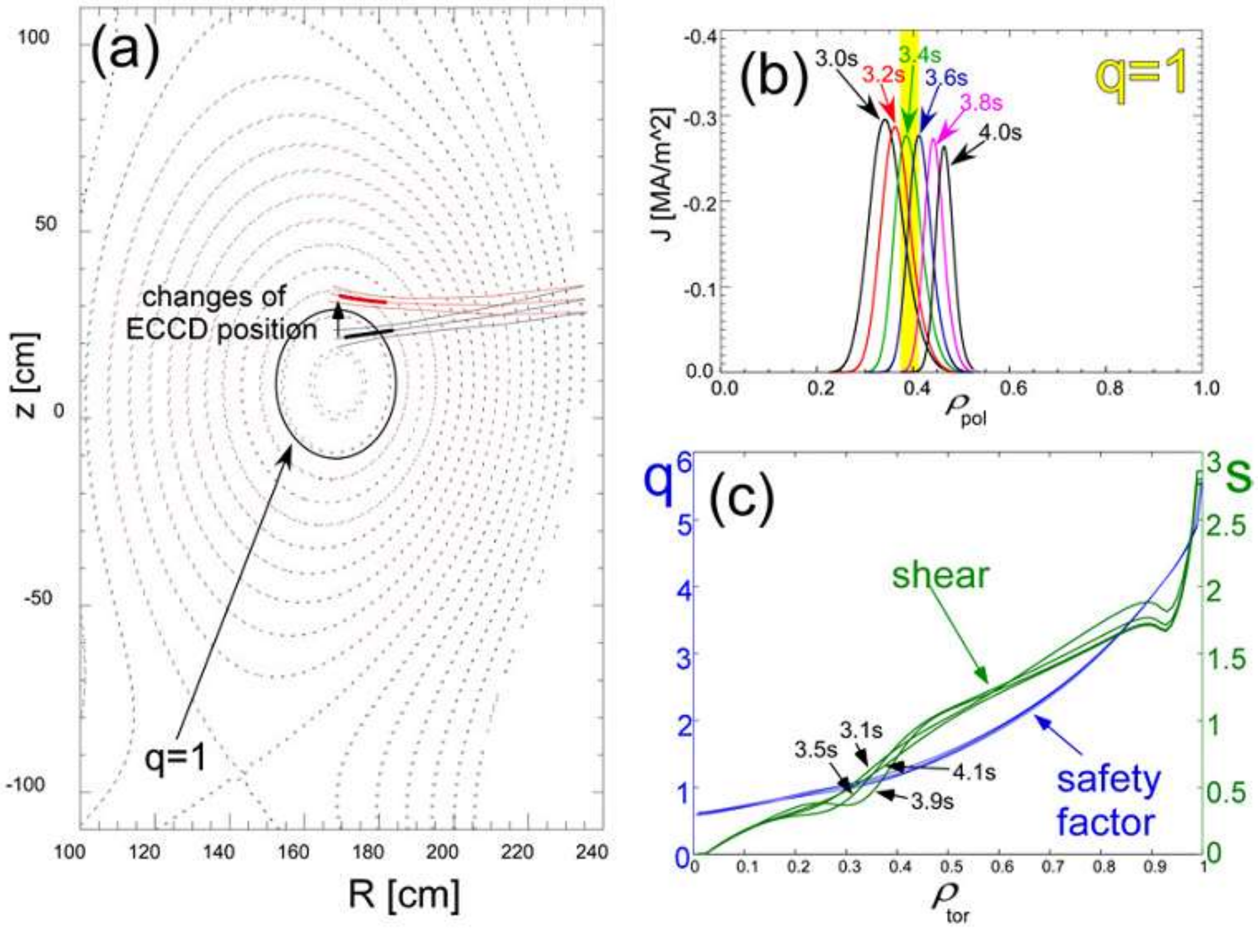
Destabilization of fast particle stabilized sawteeth

ITER relevant situation:

ICRH heating creates fast particles (mock up alpha particles in ITER).

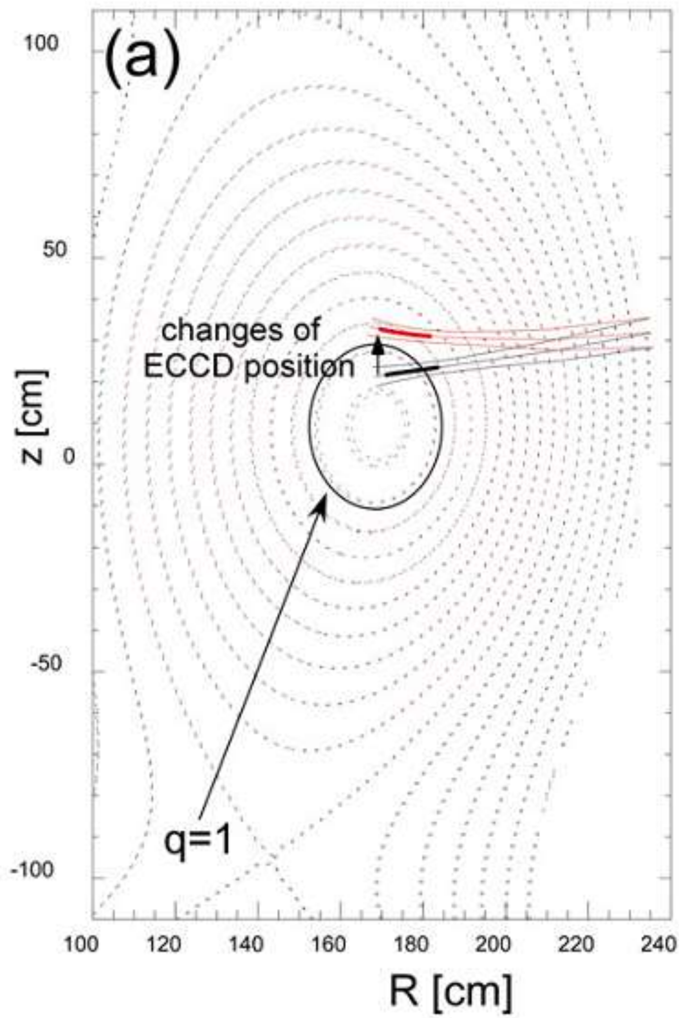
Central ECRH heating keeps the temperature profile constant (this avoids effects Te profile on the (1,1) mode stability).

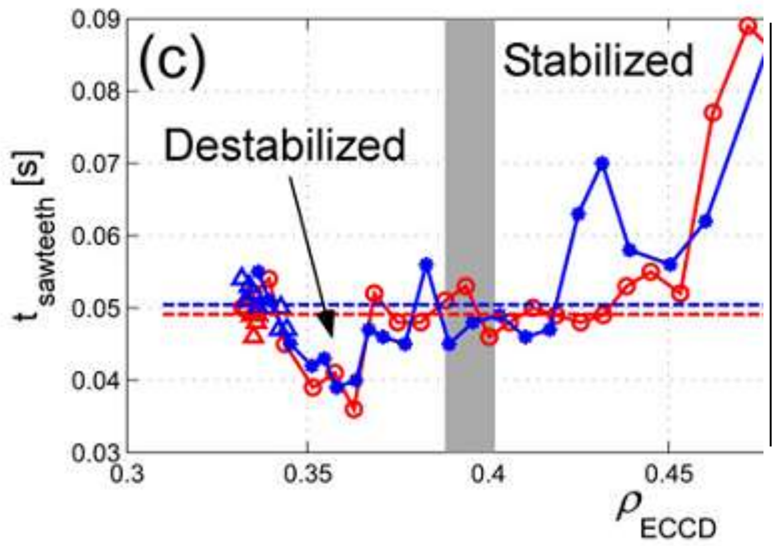
ECCD is applied around q=1 surface to change shear.



[Igochine et.al. Plasma Phys. Control. Fusion 53 (2011)]

Destabilization of fast particle stabilized sawteeth





About 40% reduction of the sawtooth period is achieved.

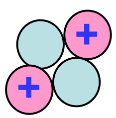
Expected sawteeth in ITER is about the critical size to trigger NTMs.

[Igochine et.al. Plasma Phys. Control. Fusion 53 (2011)]

Sawtooth Control in ITER

$$\pi \frac{\delta \hat{W}}{s_1} < c_\rho \frac{\rho}{r_1}$$

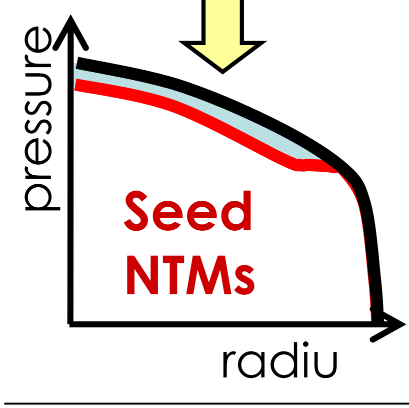
In ITER, ρ will be small and r_1 will be large



Alpha particles will stabilise the internal kink, increasing τ_{saw}

Planned control via ECCD to increase local shear

First results in presence of fast particles from ASDEX [Igochine PPCF 2011] & TORE SUPRA [Lennholm et al, PRL 2009]



Passing ions from ICRH can destabilise sawteeth [Graves et al, PRL, 2009]

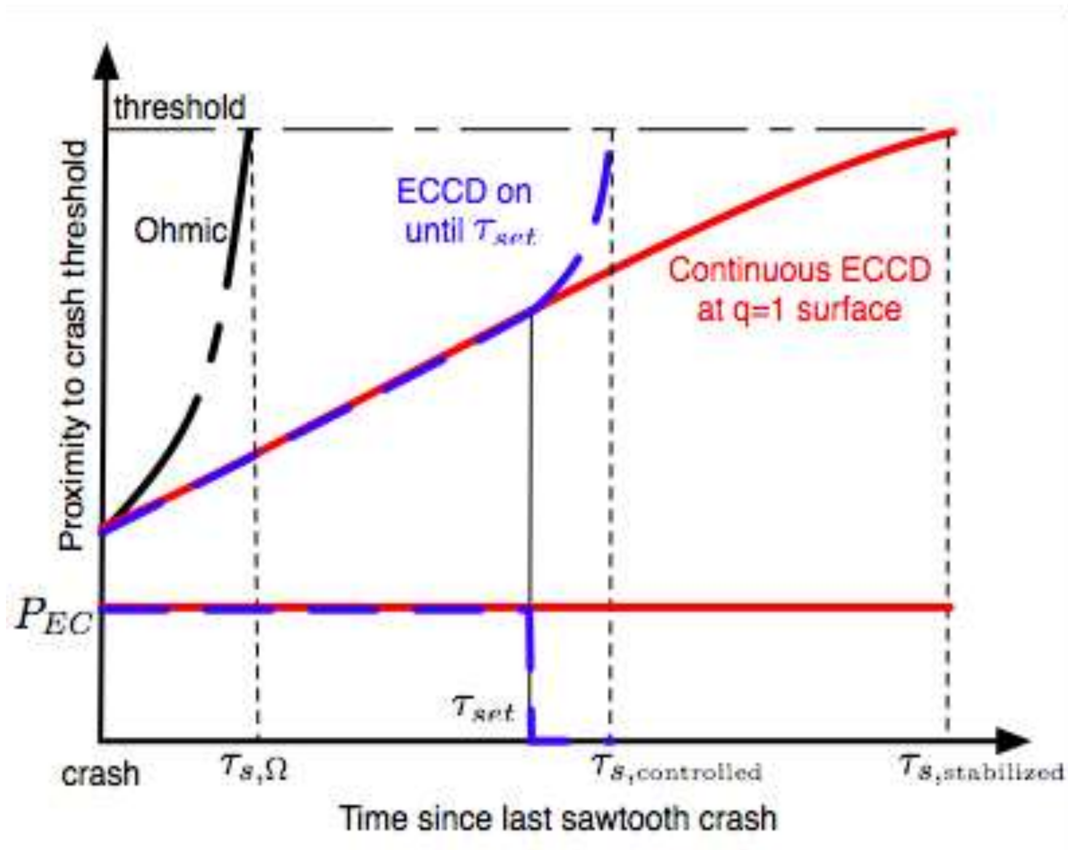
NB: He^3 minority ICRH in ITER will not drive sufficient current, but kinetic effects will help

Novel methods for sawtooth control, variant 1

Sawtooth period can be controlled by removing the stabilizing effect during the sawtooth ramp

Switch power of stabilizing actuator OFF after given time

- Crash threshold reached soon after
- Can obtain period in between natural and stabilized period
- TCV case: actuator: ECCD close to q=1, threshold: critical shear



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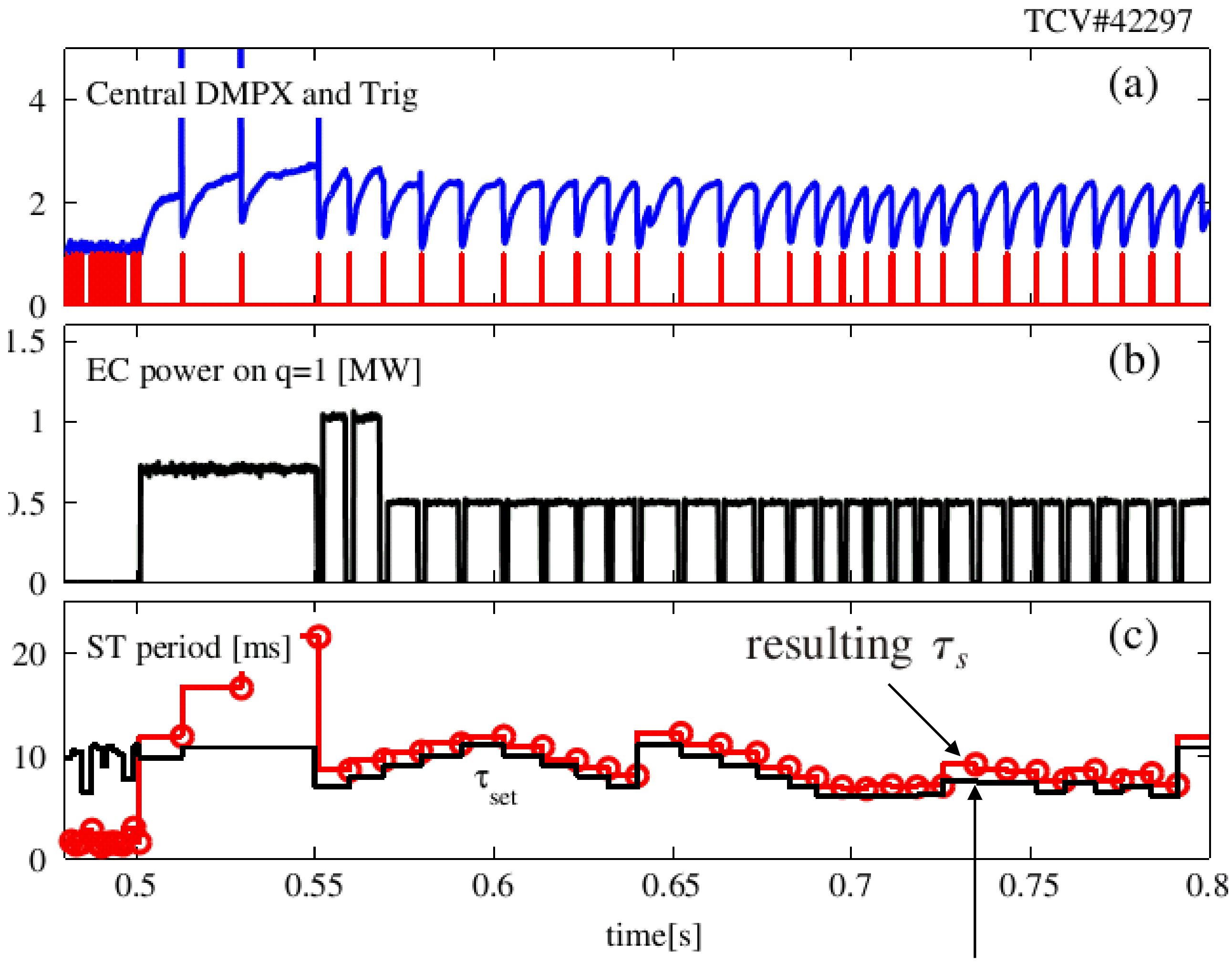


T. P. Goodman PRL **106,** 245002 (2011)

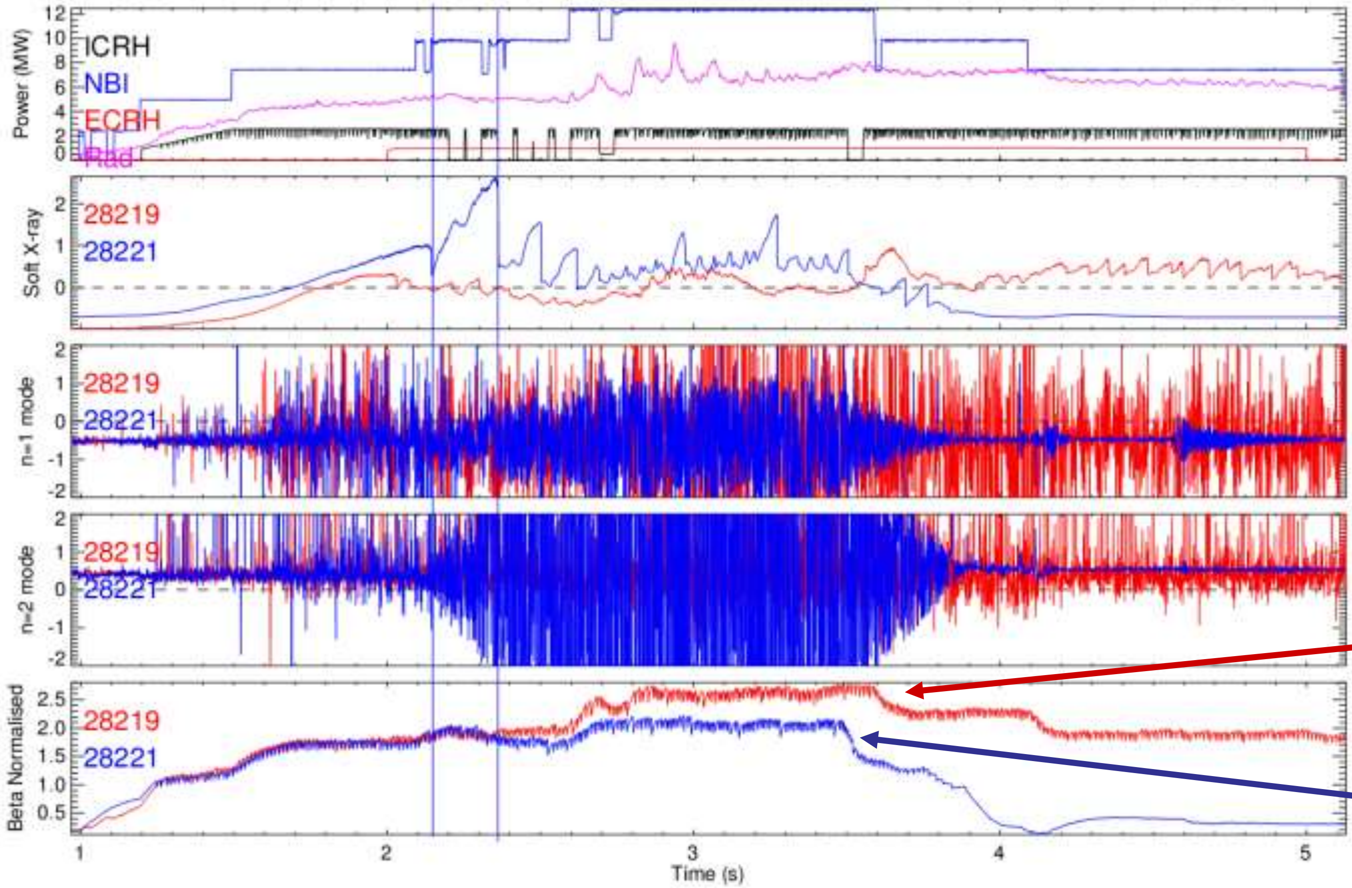
Novel methods for sawtooth control, variant 2

ASDEX Upgrade
Recent results (2012)

Is a passive action sufficient to avoid NTM?

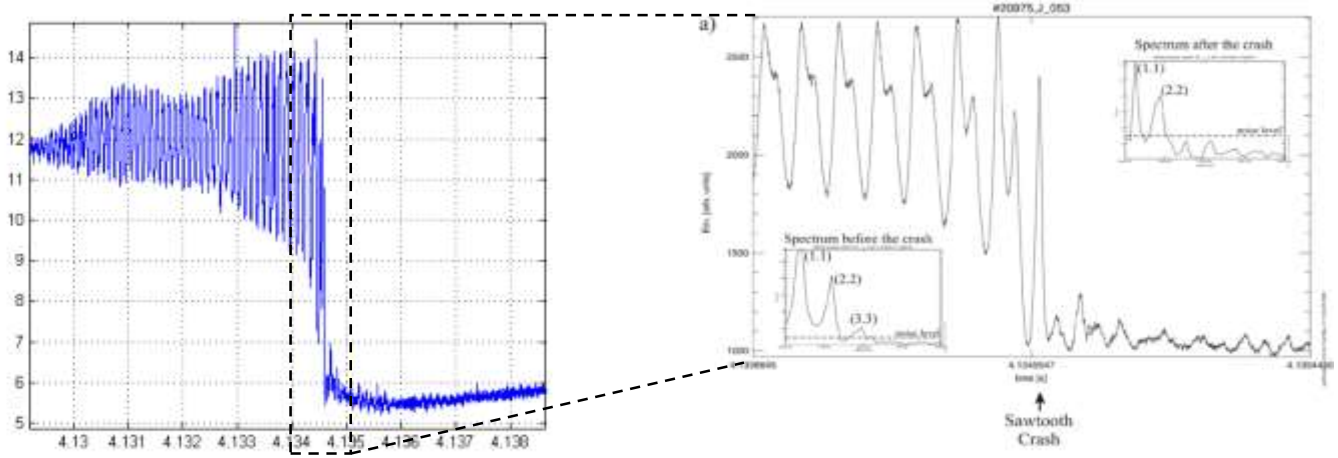
Yes, if we drive small current inside q=1.

(No precise position is required in contrast to NTM case!)



No NTM with 1MW gyrotron

NTM drops β_N by 25%!



Understanding of the sawteeth

Slow time scale

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Especially important for ITER

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- mechanism of the crash

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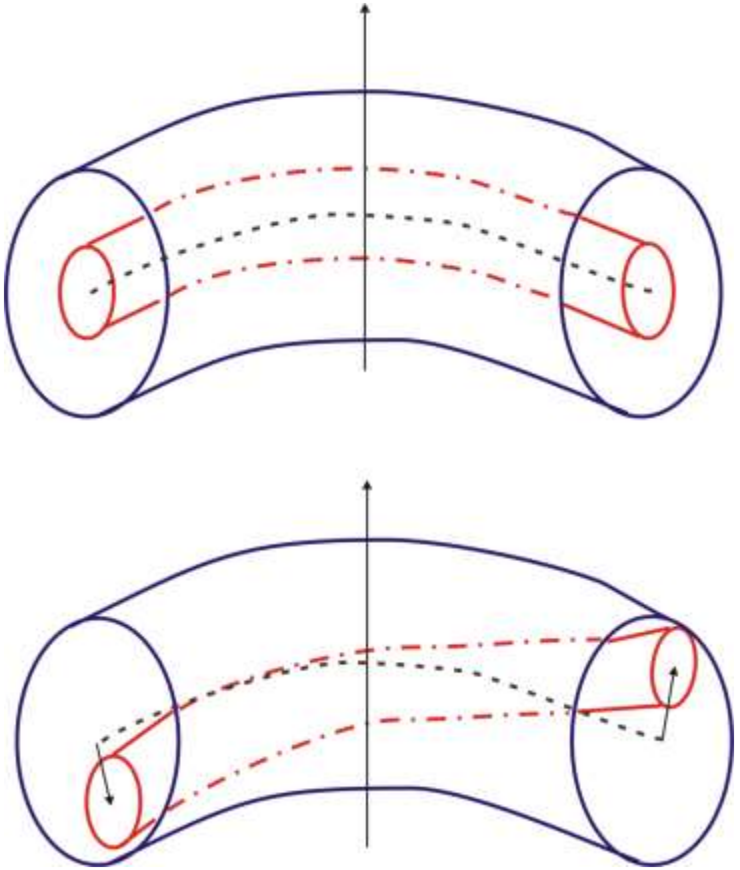
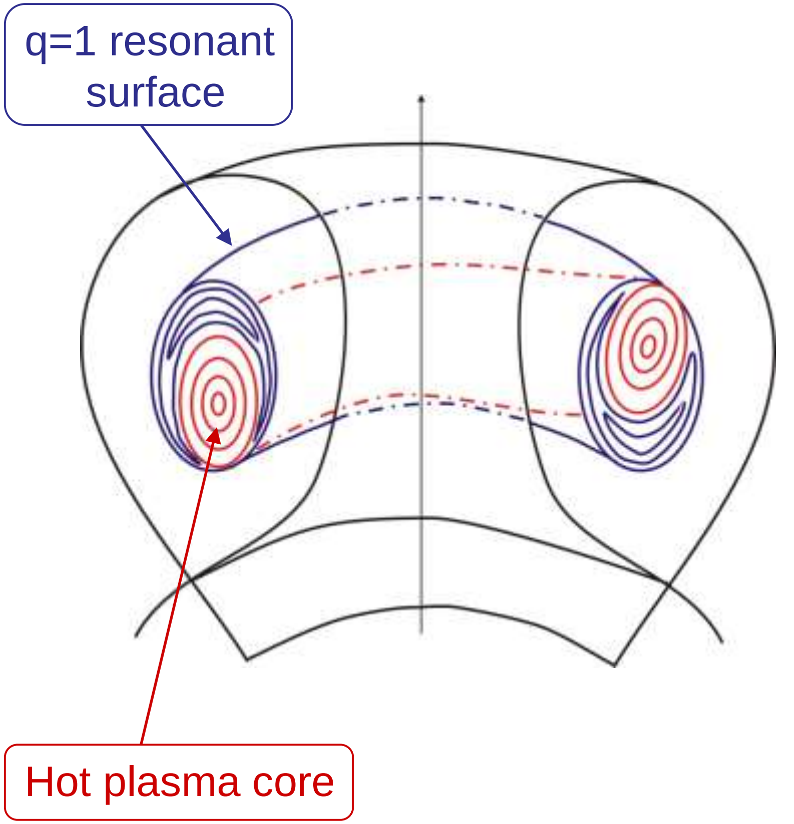
Internal Kink instability in a Tokamak.



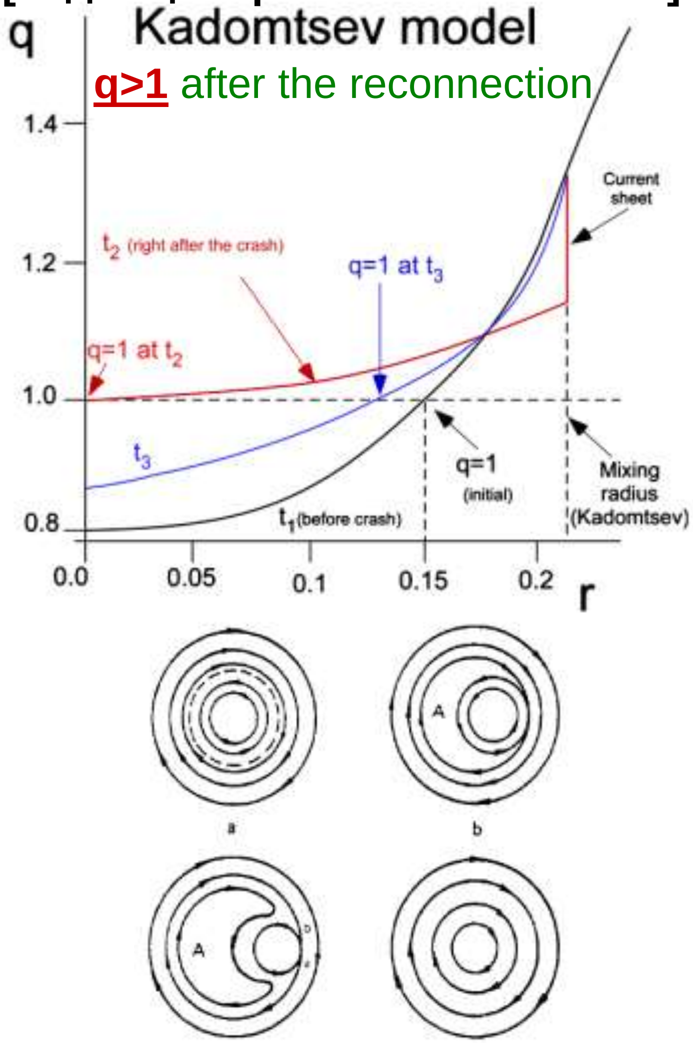
A Tilt and Shift of the Core Plasma.

Sawteeth: internal (1,1) kink mode.



Sawtooth crash (fast time scale)

[Кадомцев физика плазмы1975]

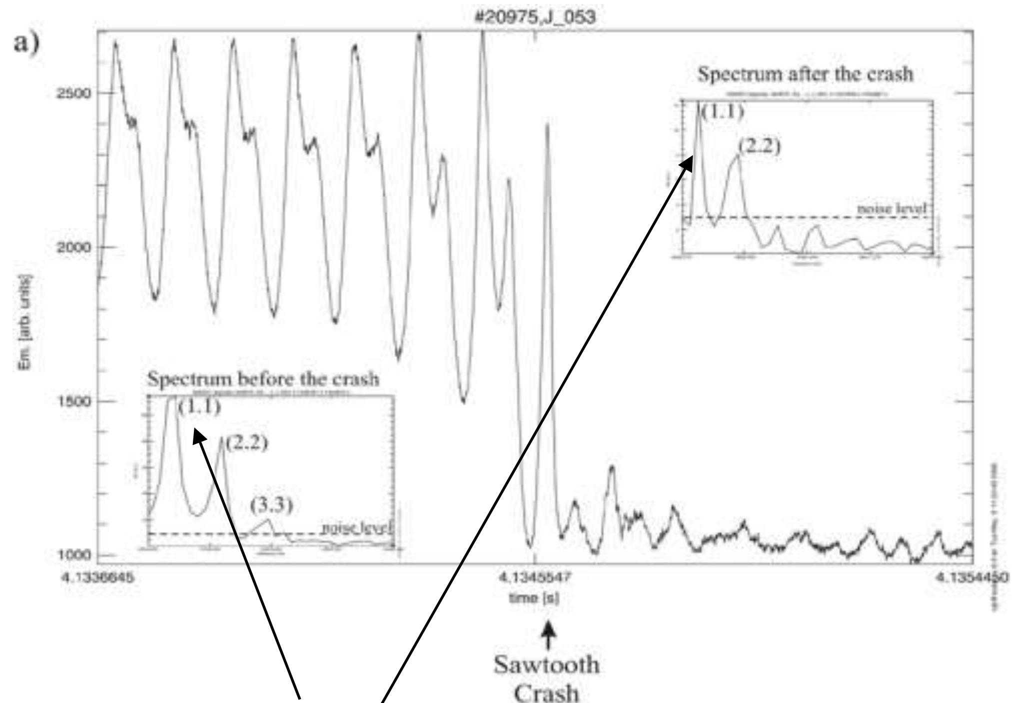


O-point becomes the new plasma center

ASDEX Upgrade discharge

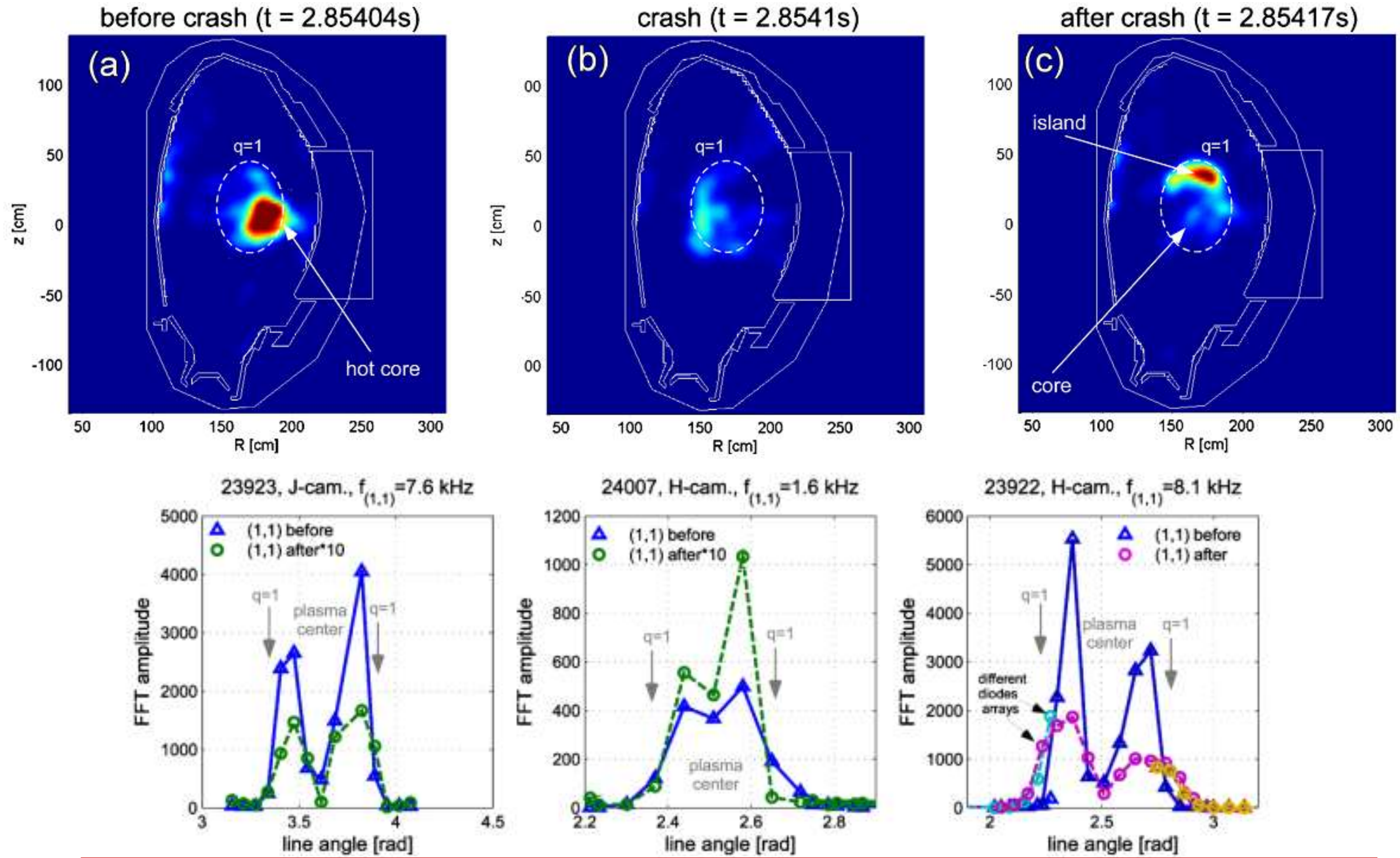
q<1 after the reconnection

[Igochine et. al., NF 2007]



(1,1) mode before and after the crash displayes existence of q<1 in the plasma after the crash

Sawtooth crash (fast time scale)



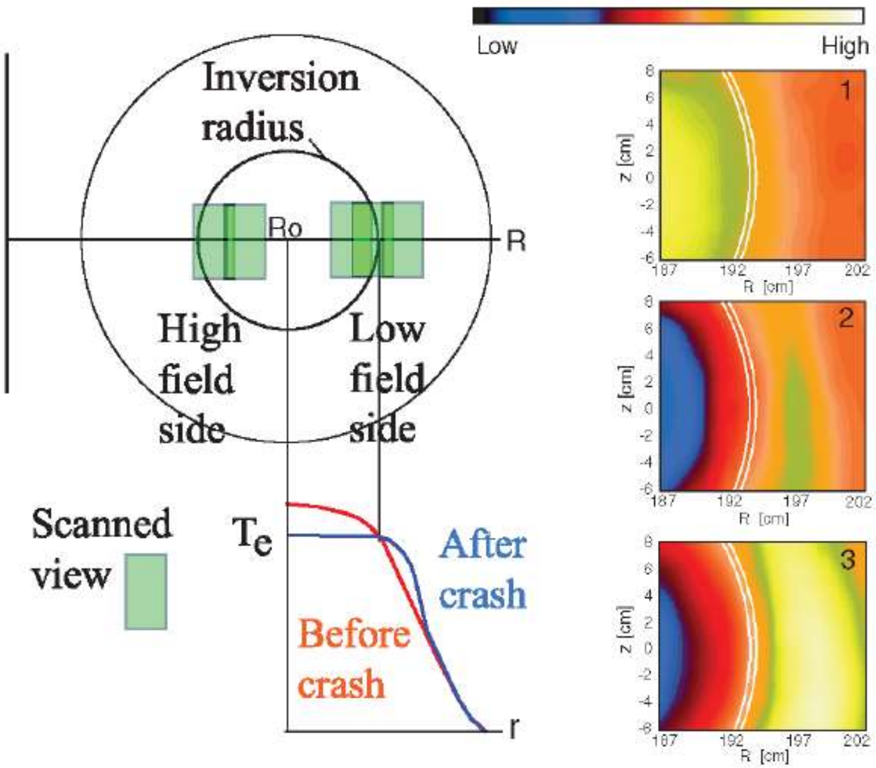
Position of (1,1) mode is the same before and after the crash!

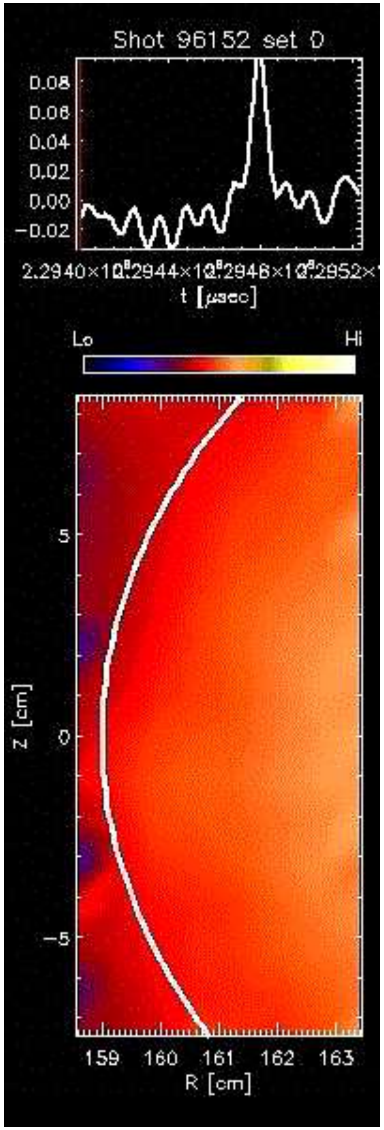
[Igochine et.al. Phys. Plasmas 17 (2010)]

Sawtooth crash in TEXTOR

High field side reconnection in TEXTOR

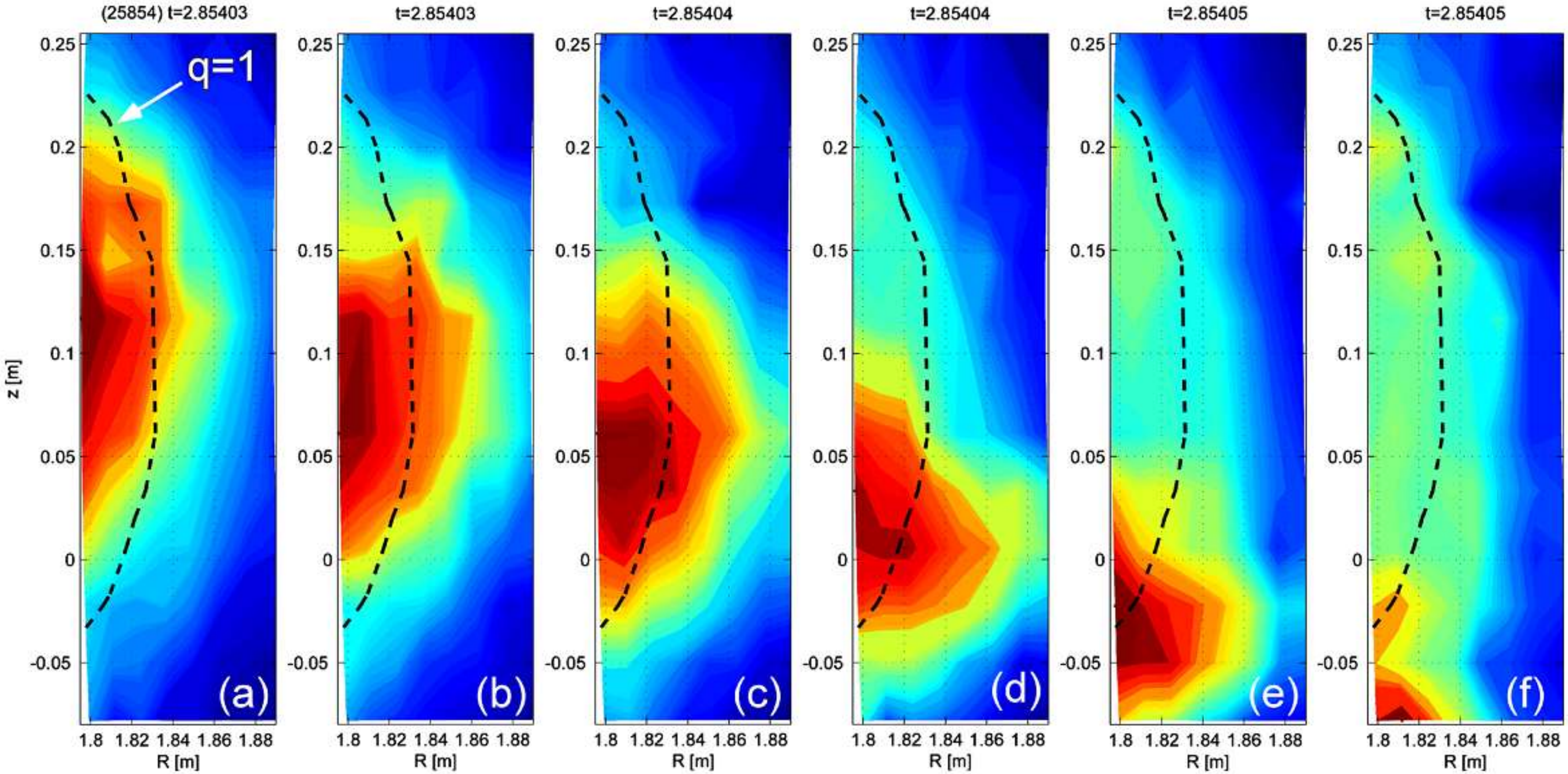
[Park, PRL and PoP 2006]





1. High field side reconnection
2. Poloidaly localized reconnection

Sawtooth crash in ASDEX Upgrade



Local outflow of the hot core through the X-point. The same as in TEXTOR.

[Igochine et.al. Phys. Plasmas 17, (2010)]

Main experimental evidences for the crash phase

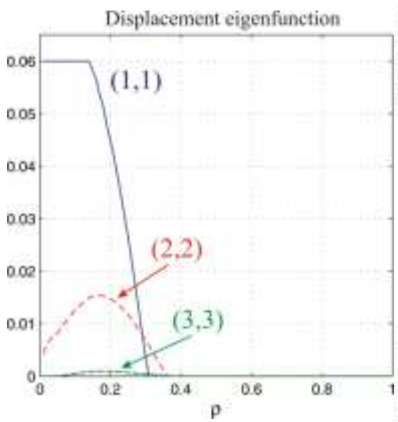
The sawtooth crash model has to fulfilled the following conditions:

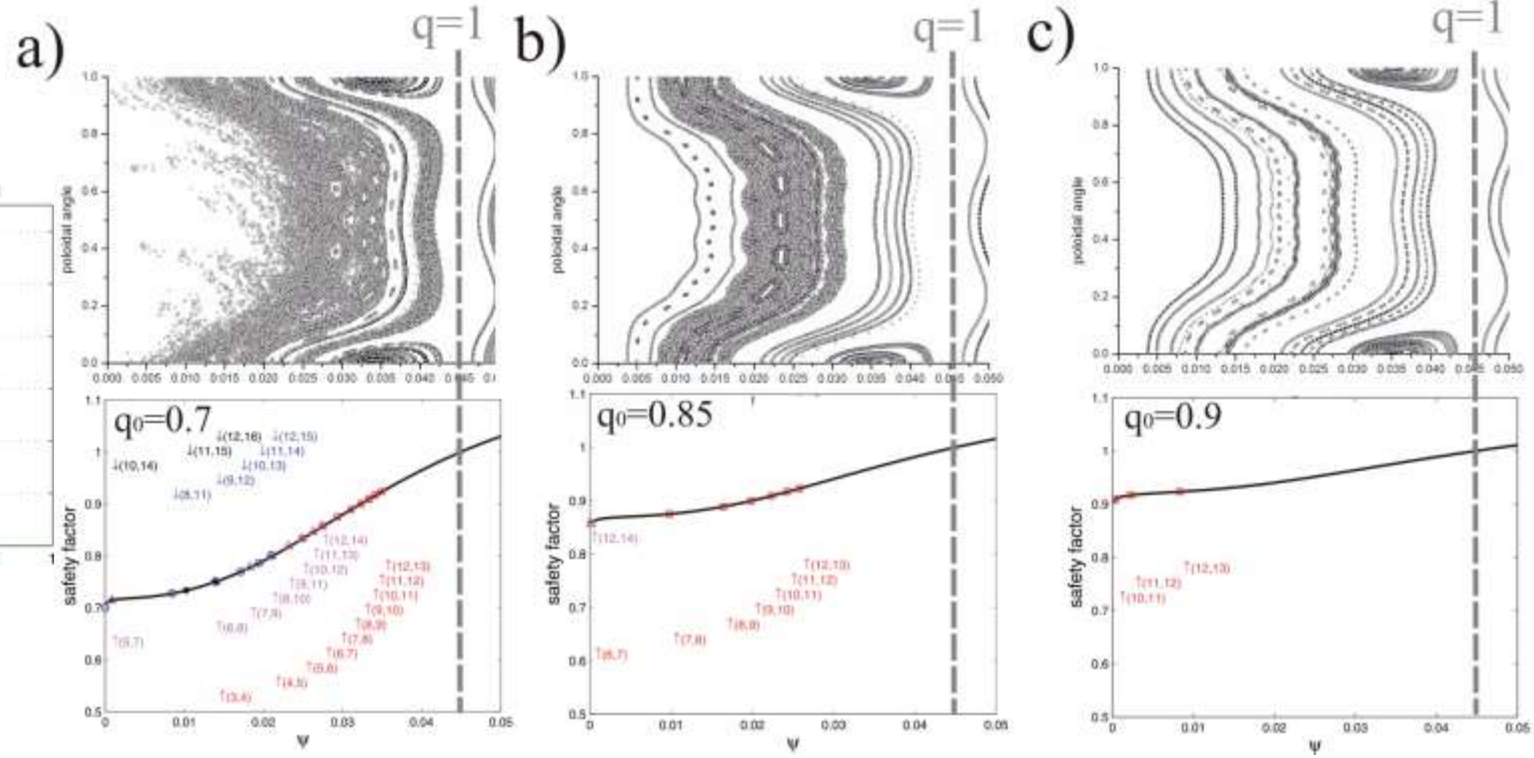
1. (1,1) mode remains after the crash
2. Position of the mode is not affected by the crash
3. Temperature from the core is removed
4. Heat outflow is local
5. Heat outflow is fast
6. Reconnection could happened in any poloidal locations, also on the high field side

Kadomtsev model **[Кадомцев физика плазмы1975]** contradicts to 1,2,5
Baloonning model **[Nishimura PoP 1999]** contradicts to 6
….
Stochastic model could be a possible explanation
Main idea: interaction of the modes leads to stochastisity which removes heat from the core and keep the mode at its original position.

Stochasticity is a result of interaction of primary (1,1) mode with harmonics (2,2) and (3,3)

ECE + SXR



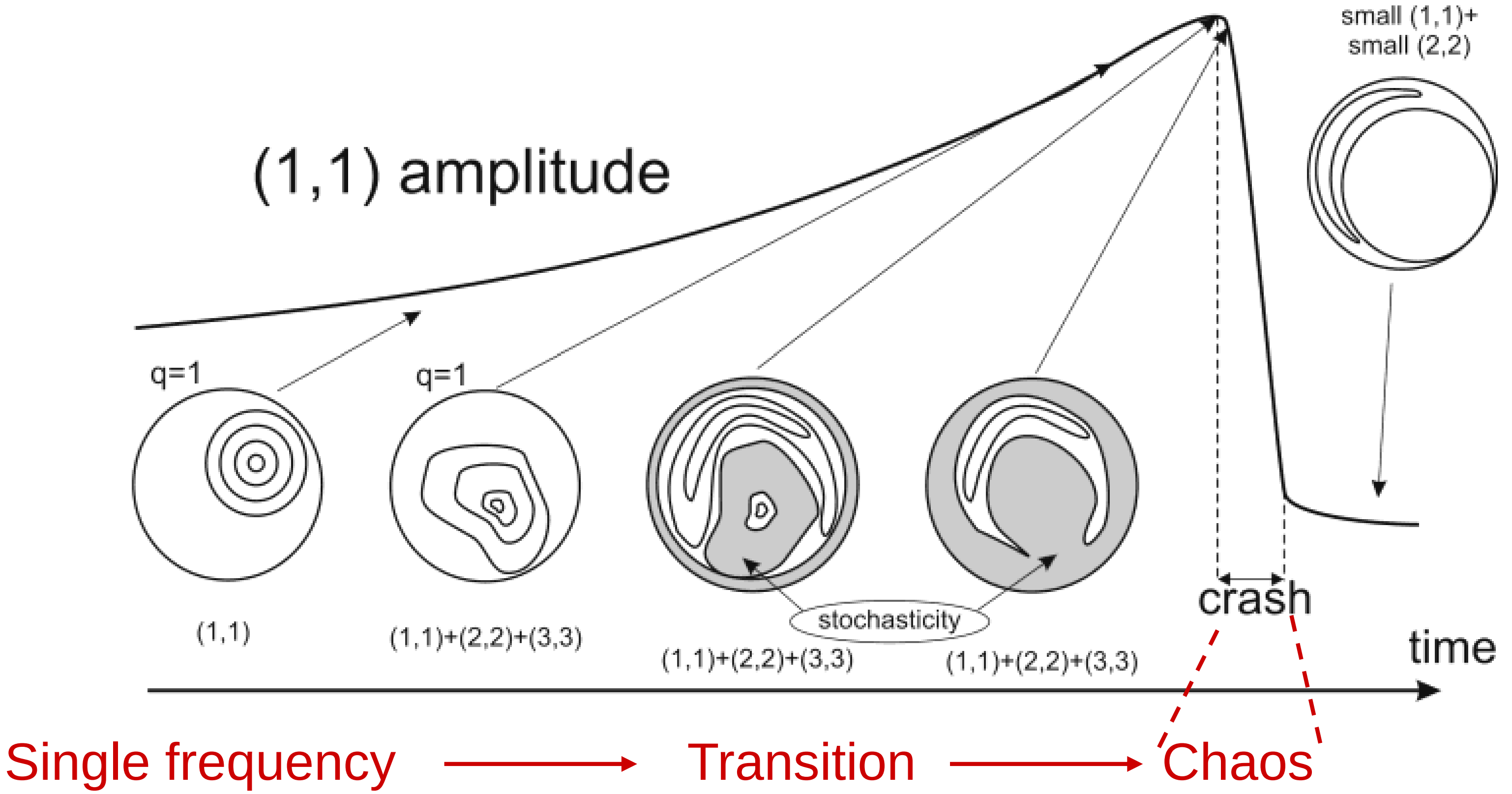


[Igochine et. al., NF 2007]

<u>Conclusions:</u>

- Experimental perturbations are sufficient to stochastize the region
- Stochastization requires the existence of several low-order rational surfaces
- q_0 determines the number of such surfaces → the main factor is q_0!

Field line tracing assumes static problem. All perturbations are coupled by definition. What could we say about dynamics of the process?



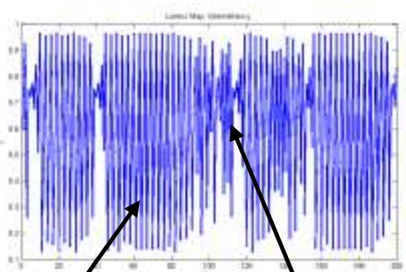
Possible variants of the transition to chaos

Hilborn “Chaos and Nonlinear Dynamics”

Roads to chaos

Intermittency

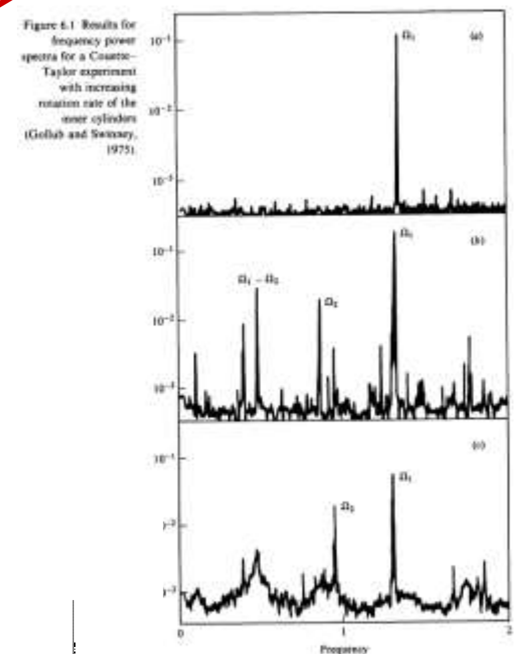
signal



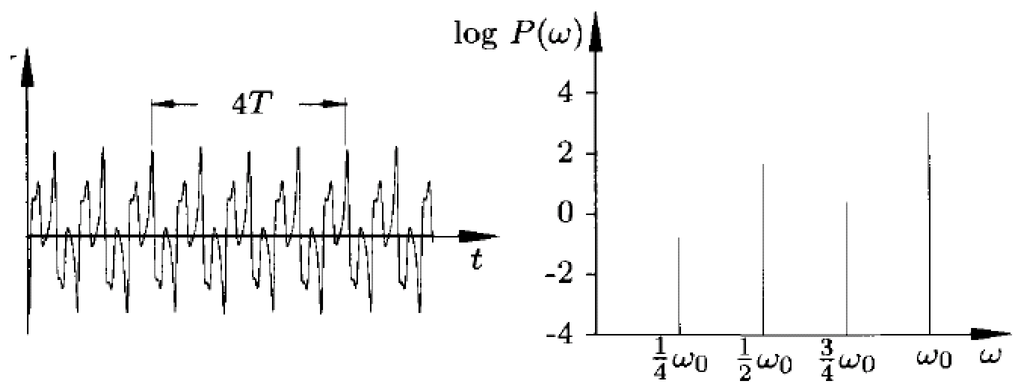
Regular phase

Stochastic phase

Quasi-periodicity



Period doubling



<u>Physical, chemical, biological systems use one of these variants.</u>
<u>Roads to chaos are universal!</u>

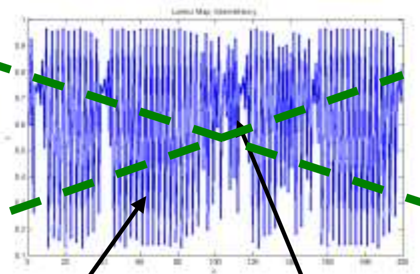
Possible variants of the transition to chaos and sawtooth crash

Hilborn “Chaos and Nonlinear Dynamics”

Roads to chaos

Intermittency

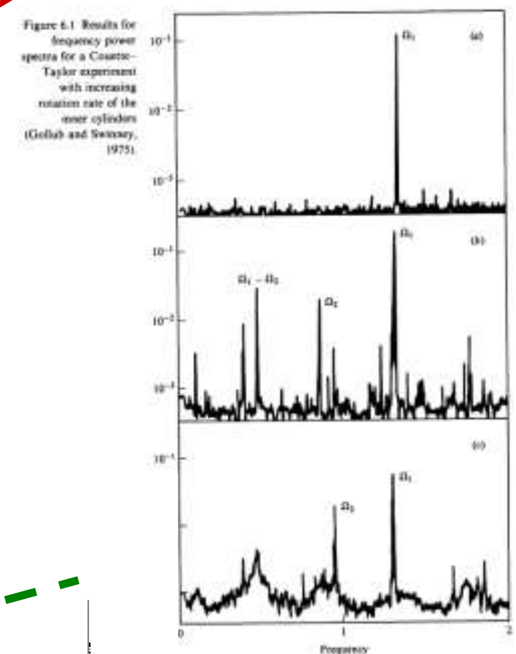
signal



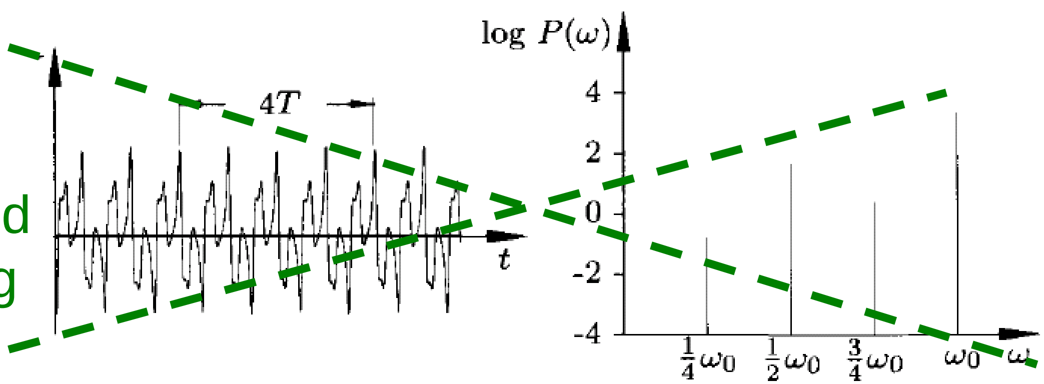
Regular phase

Stochastic phase

Quasi-periodicity



Period doubling



Sawtooth show
no intermittency and
no period doubling

Possible candidate is only quasiperiodicity

Observation of quasiperiodic transition in a cristal

Single freq. with harmonics

Quasiperiodic phase

Chaos

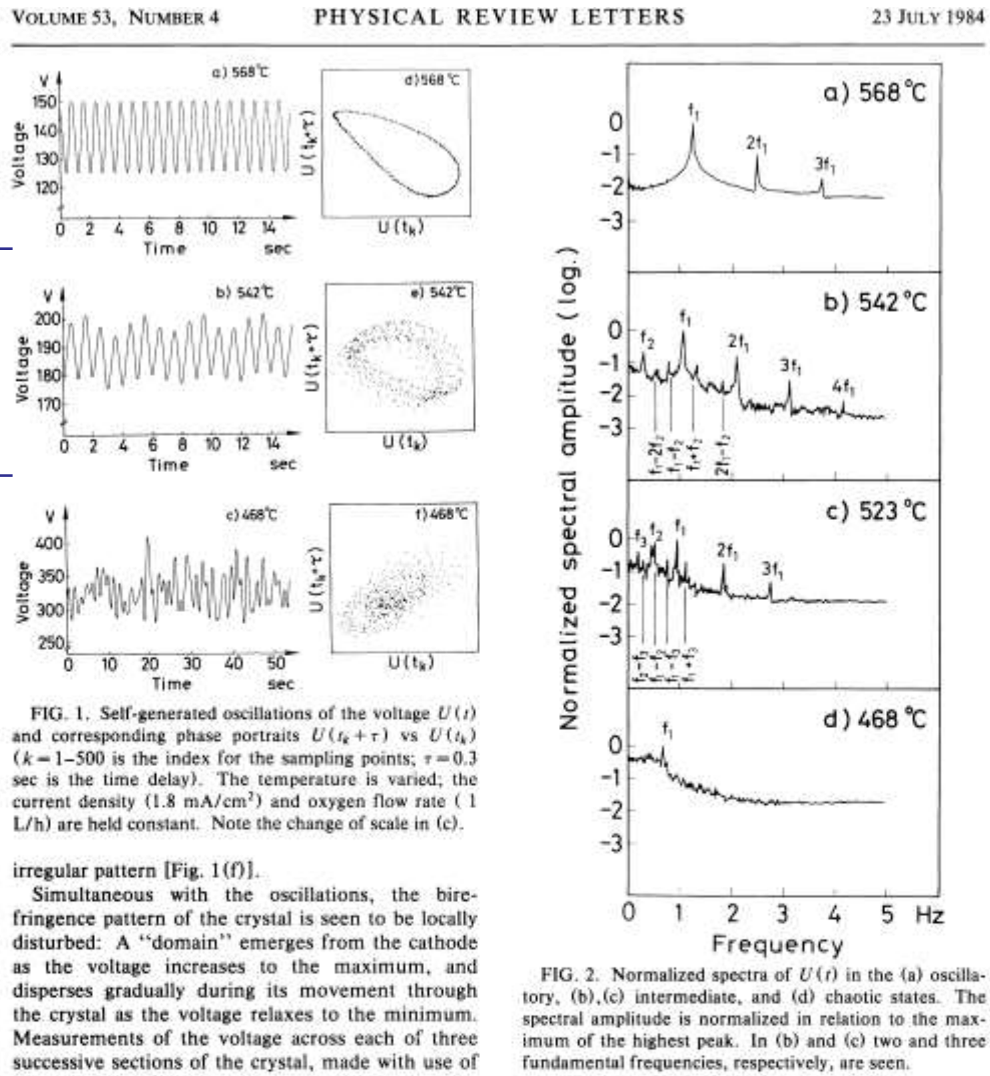

VOLUME 53, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 1984

FIG. 1. Self-generated oscillations of the voltage $U(t)$ and corresponding phase portraits $U(t_k+\tau)$ vs $U(t_k)$ ($k=1$–500 is the index for the sampling points; $\tau=0.3$ sec is the time delay). The temperature is varied; the current density (1.8 mA/cm^2) and oxygen flow rate (1 L/h) are held constant. Note the change of scale in (c).

irregular pattern [Fig. 1(f)].

Simultaneous with the oscillations, the birefringence pattern of the crystal is seen to be locally disturbed: A "domain" emerges from the cathode as the voltage increases to the maximum, and disperses gradually during its movement through the crystal as the voltage relaxes to the minimum. Measurements of the voltage across each of three successive sections of the crystal, made with use of intermediate platinum wire loop electrodes, show

FIG. 2. Normalized spectra of $U(t)$ in the (a) oscillatory, (b),(c) intermediate, and (d) chaotic states. The spectral amplitude is normalized in relation to the maximum of the highest peak. In (b) and (c) two and three fundamental frequencies, respectively, are seen.

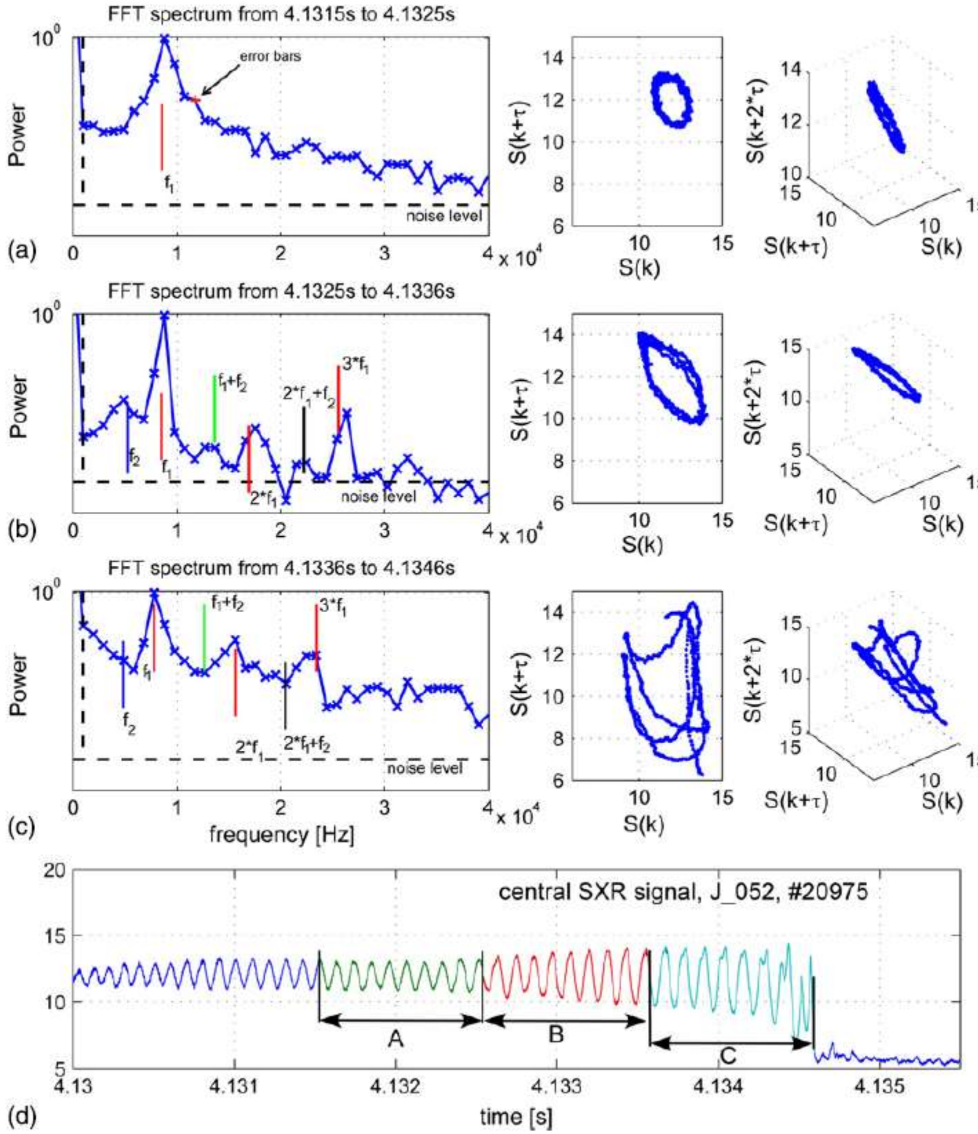
Single freq. with harmonics

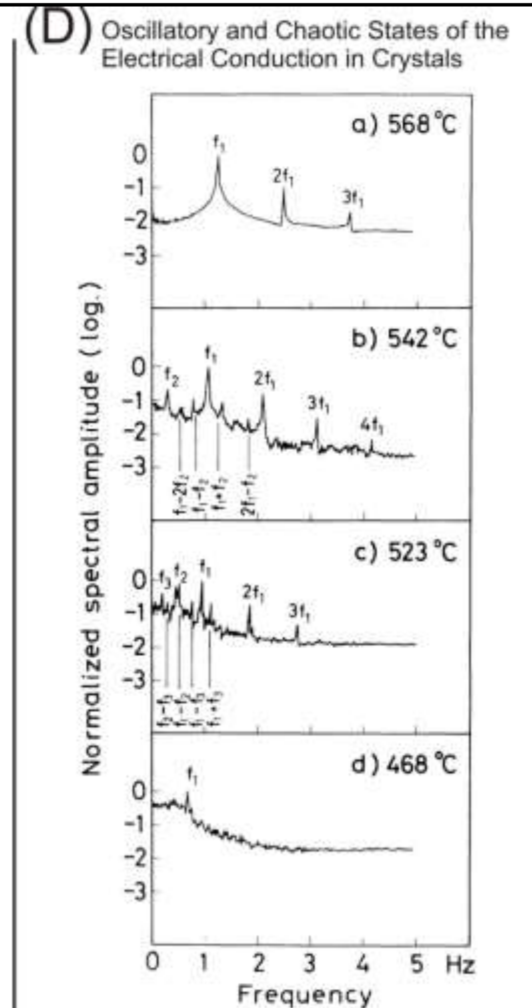
Quasiperiodic phase.
Two incommensurable frequencies and their linear combinations

Chaos

[S.Martin et.al., PRL, 1984]

Transition to chaos from central SXR signal

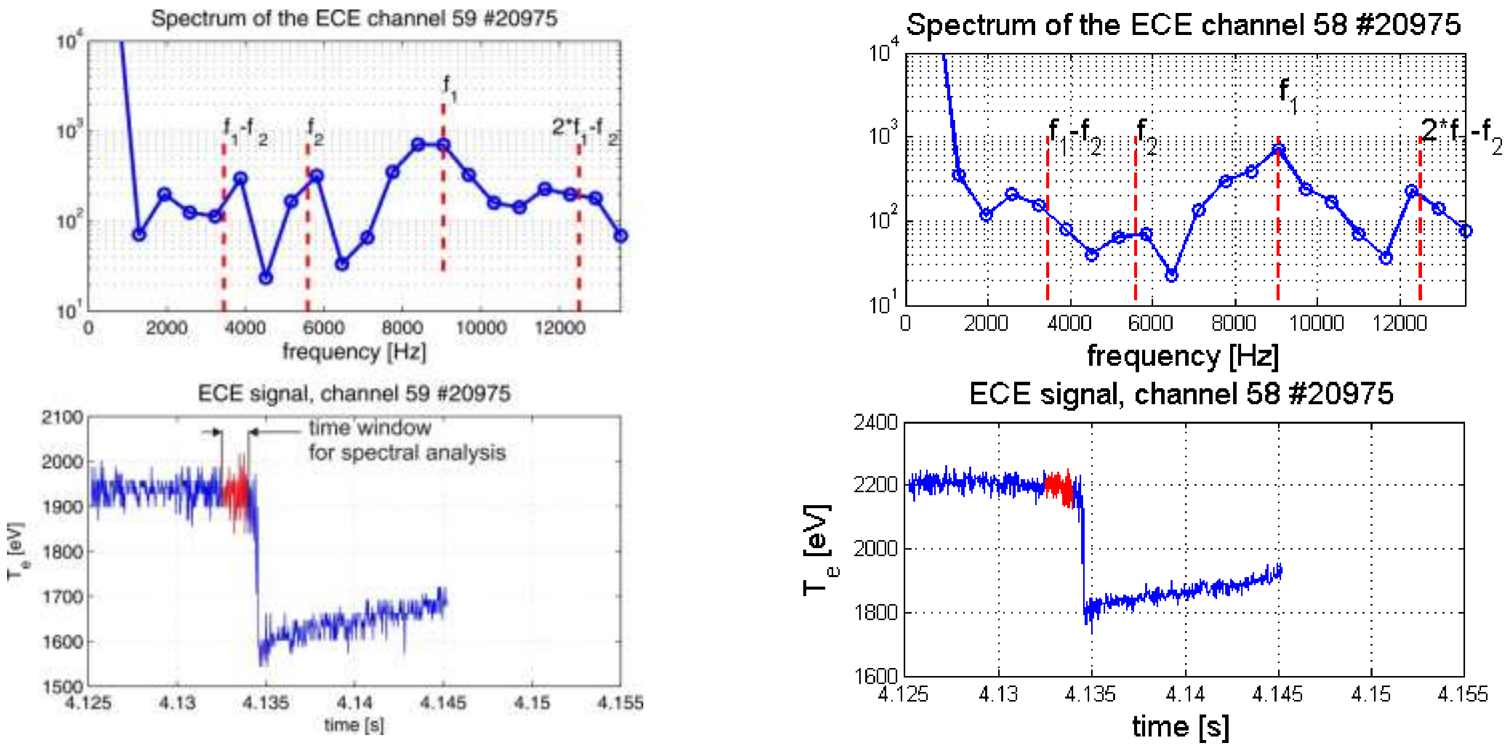




Normalized spectra of $U(t)$ in the (a) oscillatory, (b),(c) intermediate, and (d) chaotic states. The spectral amplitude is normalized in relation to the maximum of the highest peak. In (b) and (c) two and three fundamental frequencies, respectively, are seen.

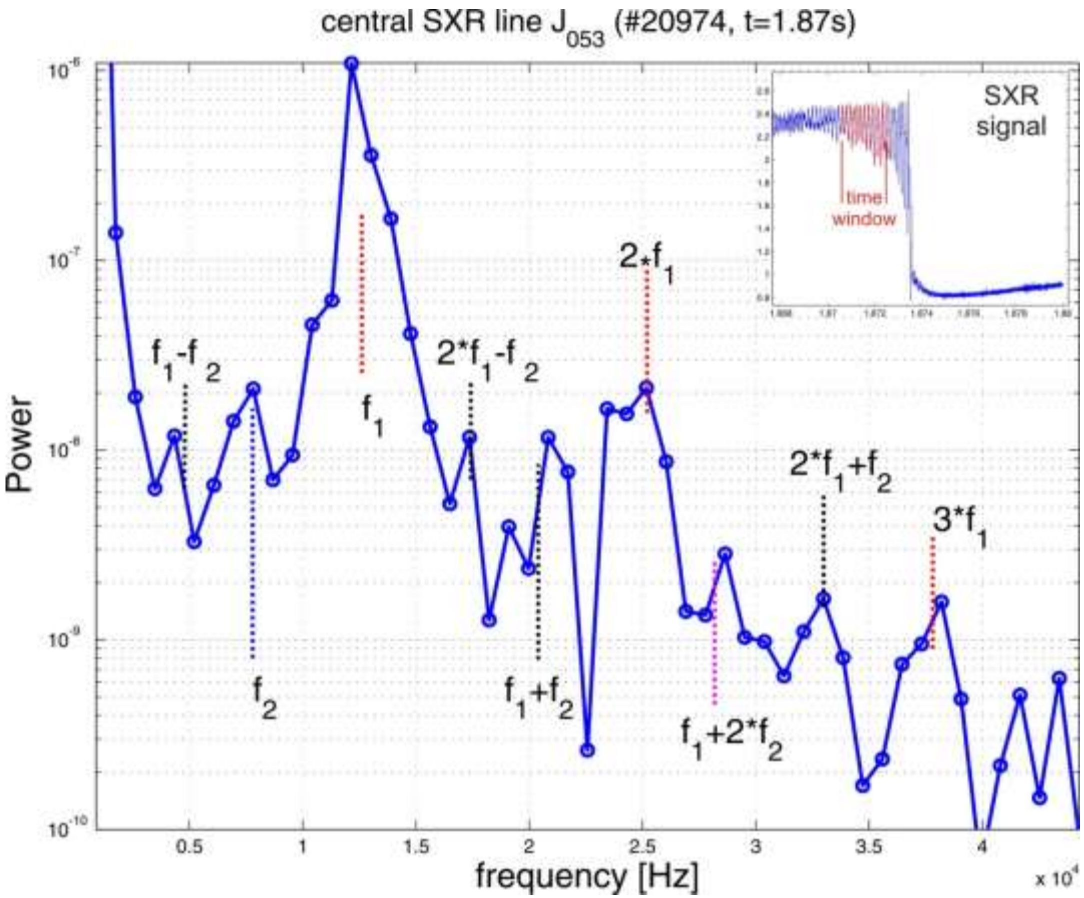
Transition to chaos from central ECE signal

ECE power spectrum and the signal for the same sawtooth crash from two central ECE channels



These measurements resolve also other combinations of the resonances which are not seen in SXR signals

Transition to chaos from central SXR signal



Other discharge show the same spectrum in slightly quasiperiodic stage.

The low frequency spectrum is completely described by two primary frequencies and their linear combinations.

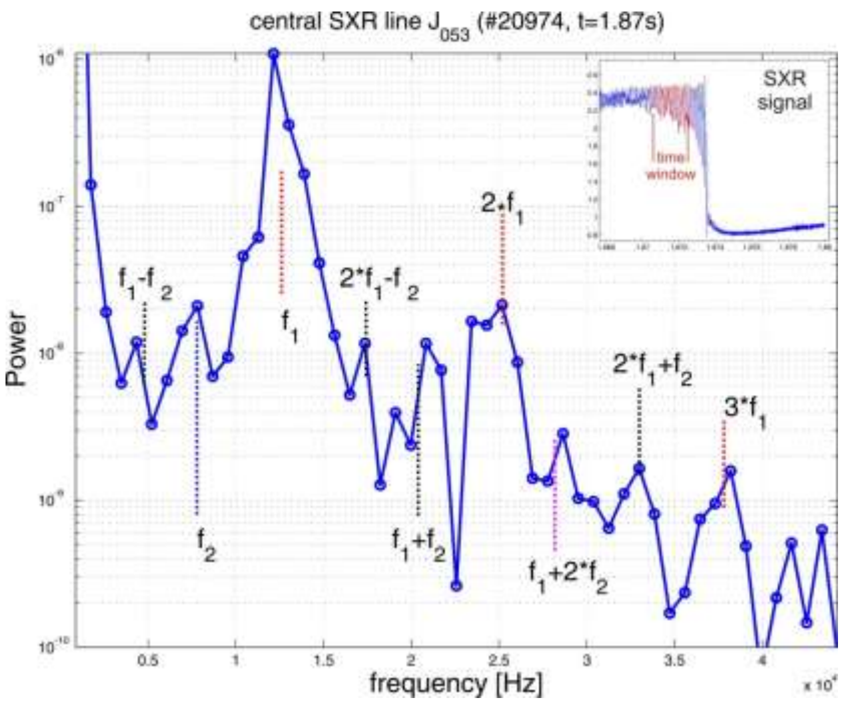
Relation between two frequencies is golden mean ("the most irrational number")

$$\frac{f_2}{f_1} = \frac{\sqrt{5}-1}{2} \approx 0.618$$

<u>This fact indicates that the chaos in the system is approached in the most "intense" way</u>

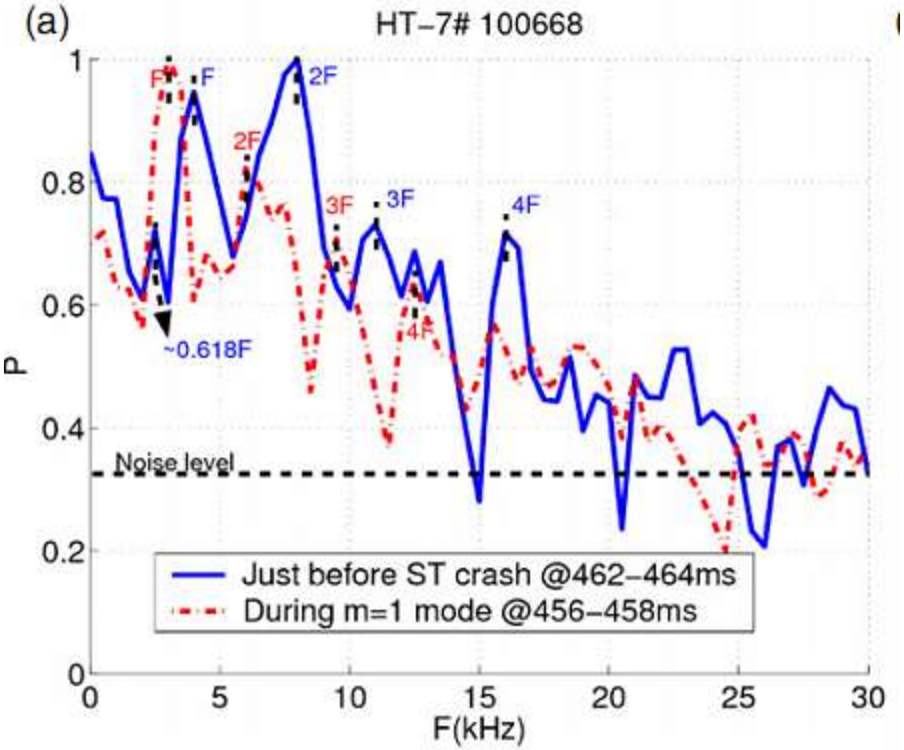
[Igochine et.al. Nucl. Fusion 48 (2008)]

ASDEX Upgrade

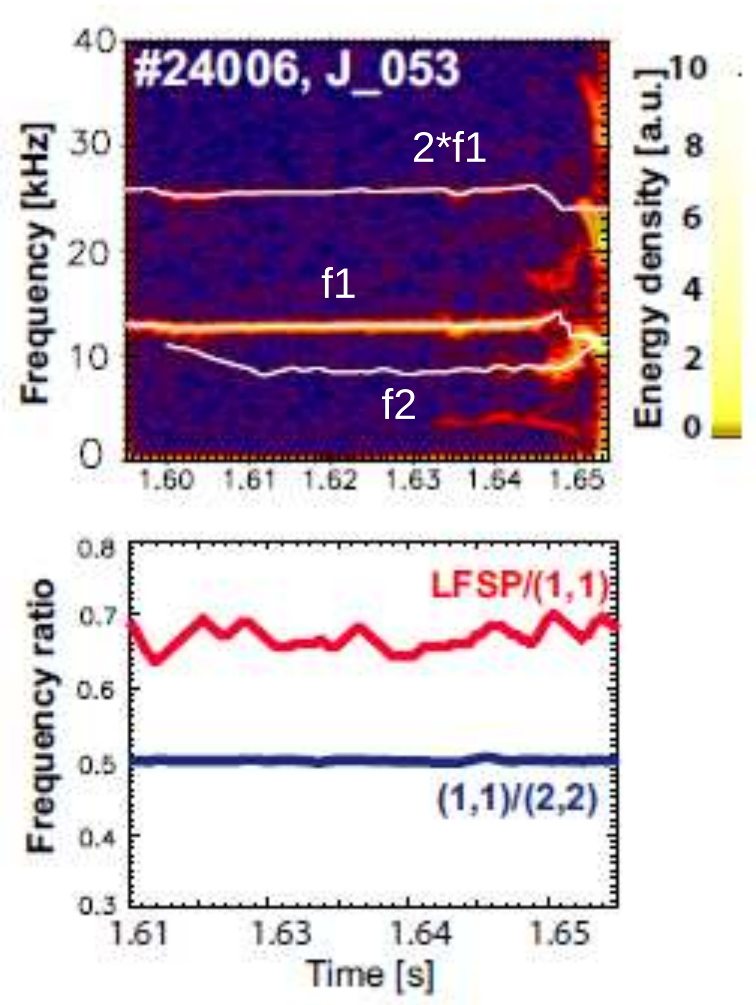


[Igochine et.al. Nucl. Fusion 48 (2008)]

HT-7



[Sun et.al. Plasma Phys. Control. Fusion 51 (2009)]



More detail investigation of a large set of sawtooth crashes (wavelet technique, coherence technique and bi-coherence technique) :

-Confirm existence of the mode with golden mean relation to the primary (1,1)

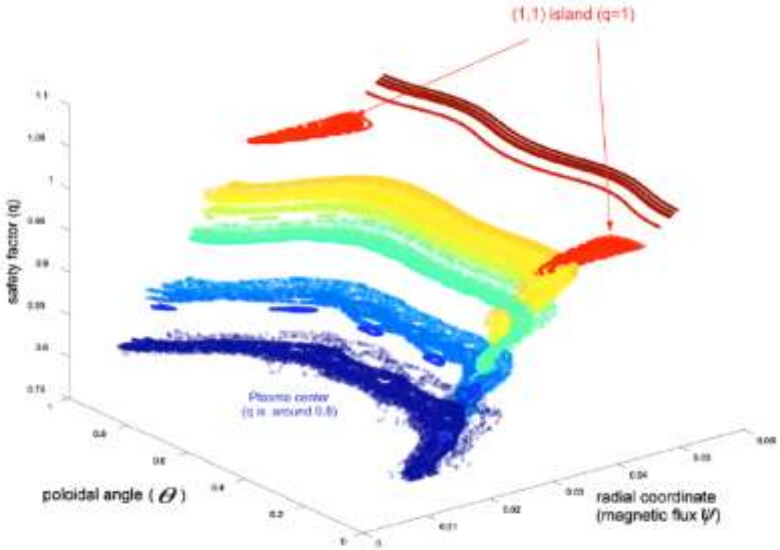
- Give out typically (1,1) mode structure for this mode as well

- Long precursor phase is required to identify the second mode!

[Papp et.al. Plasma Phys. Control. Fusion 53 (2011)]

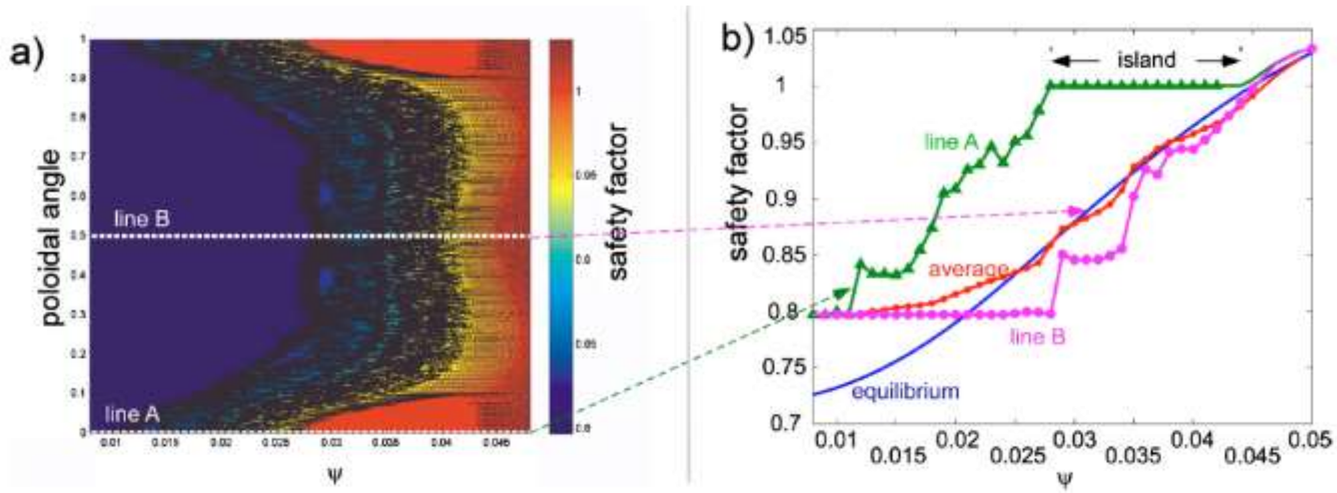
Changes of the safety factor during the crash in stochastic model

[Igochine et.al. Phys. Plasmas 17 (2010)]



$$q = \lim_{\Delta\phi \to \infty} \frac{\Sigma \Delta \phi}{\Sigma \Delta \theta} \approx \frac{2\pi \cdot (N-1)}{\Sigma_{i=1}^{N-1} (\theta_{i+1} - \theta_i)}$$

Safety factor remains almost the same in stochastic model (contrary to Kadomtsev model!)

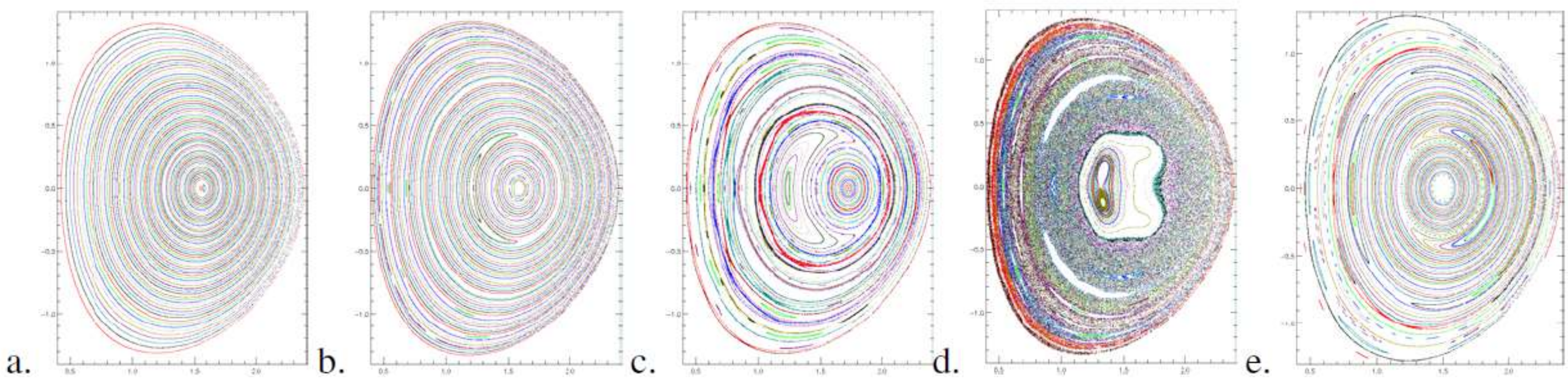


Sawtooth modelling

- Nonlinear MHD simulations (M3D code) show stochastisity.

- but .. „multiple time and space scales associated with the reconnection layer and growth time make this an extremely challenging computational problem. … and there still remain some resolution issues.”

Small tokamak → small Lundquist number: $S = 10^4$ (big tokamaks 10^8)
Lundquist number = (resistive diffusion time)/(Alfven transit time)



[Breslau et.al. Phys. Plasmas 14, 056105, 2007]

Non-linear simulations of the sawtooth is very challenging task (even in a small tokamak).

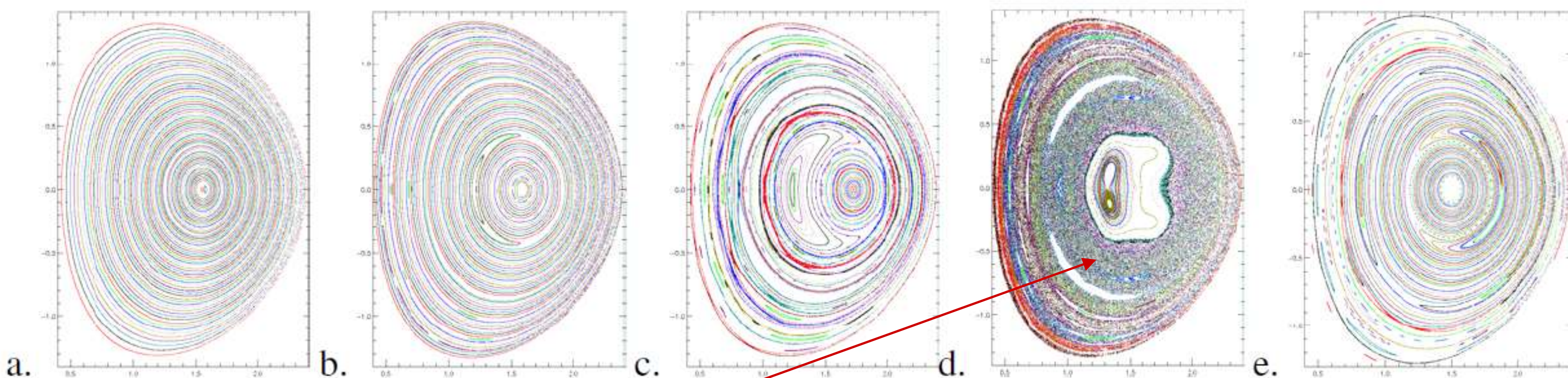
Sawtooth modelling

Ohm's law, 2 fluid MHD

$$\underbrace{\eta \vec{j}}_{\text{resistive MHD}} = \underbrace{\vec{E} + \vec{v}\times\vec{B}}_{\text{ideal MHD}} - \underbrace{\frac{1}{en_e}\vec{j}\times\vec{B}}_{\text{Hall term}} + \underbrace{\frac{1}{en_e}\nabla p_e}_{\text{electron pressure term}}$$

…at least two fluid MHD with correct electron pressure description are necessary for reconnection region (fast crash time, smaller stochastic region)!

<u>We will discuss this point in details in the last lecture</u>



[Breslau et.al. Phys. Plasmas 14, 056105, 2007]

Stochastic region is too large,… much more then visible in the experiments (heat outflow is rather global instead of local as in the experiments)

Summary of the stochastic model

<u>Stochastic model explains:</u>

- Constant position of the q=1 surface during the crash (1,1)
- Fast remove of the heat from the core
- It has no contradiction with observations
- Indications of the transition to stochastic phase are found

MHD calculations show stochastisity during the crash… but the *<u>degree of stochastization is too big</u>* (not all important physics is inside)

Degree of stochastization is not clear. Two situations are possible:

-Fully stochastic core

-Partially stochastic core (only along the separatrix)

<u>In both cases, (1,1) island is not destroyed in stochastic model!</u>

Conclusions

We are quite successful in controlling of sawteeth even with fast particles (ITER relevant situation).Many control techniques are developed.

During the last years a new important information about the crash phase were obtained

Stochastic model is a possible candidate for the explanation of the sawtooth crash.

There are still lack of sawtooth modeling with full nonlinear MHD codes with 2 fluid effects.

Full understanding of the crash phase is necessary for exact predictions of sawtooth amplitude and period for ITER and DEMO.

More information are here

The physics of sawtooth stabilization

I T Chapman[1], S D Pinches[1], J P Graves[2], R J Akers[1], L C Appel[1], R V Budny[3], S Coda[2], N J Conway[1], M de Bock[4], L-G Eriksson[5], R J Hastie[1], T C Hender[1], G T A Huysmans[5], T Johnson[6], H R Koslowski[7], A Krämer-Flecken[7], M Lennholm[5], Y Liang[7], S Saarelma[1], S E Sharapov[1], I Voitsekhovitch[1], the MAST and TEXTOR Teams and JET EFDA Contributors[8]

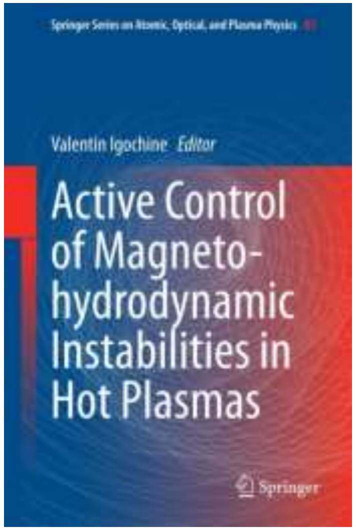
Plasma Phys. Control. Fusion **49** (2007) B385–B394

SAWTOOTH INSTABILITY IN TOKAMAK PLASMAS

R. J. HASTIE
Imperial College, London, UK

Astrophysics and Space Science **256:** 177–204, 1998.
© 1998 *Kluwer Academic Publishers. Printed in the Netherlands.*

Chapter 4 of this book, written by I.Chapman



V. Igochine, “Active Control of Magneto-hydrodynamic Instabilities in Hot Plasmas”, Springer Series on Atomic, Optical, and Plasma Physics, Vol. **83**, 2015